On independent set on B_1-EPG graphs

Marin Bougeret (Lirmm, Montpellier, France)
Joint Work with S. Bessy, D.Goncalves, C. Paul

WAOA 2015

OnIndependent set on B_1-EPG graphs

Marin Bougeret (Lirmm, Montpellier, France)
Joint Work with S. Bessy, D.Goncalves, C. Paul

WAOA 2015

On independent set on B_1-EPG graphs

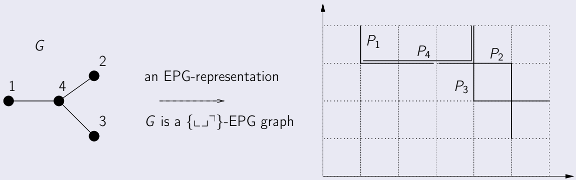
Marin Bougeret (Lirmm, Montpellier, France)
Joint Work with S. Bessy, D.Goncalves, C. Paul

WAOA 2015

Outline

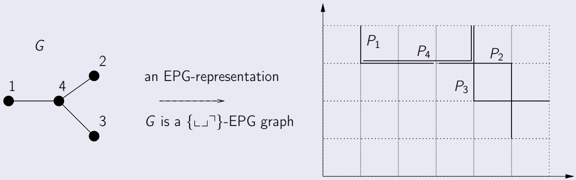
Definition of EPG-Graphs

- EPG (for Edge intersection graphs of Paths on a Grid) graphs introduced in [GLS09]
- In EPG-graph $G = (V, E)$:
  - each vertex v corresponds to a path P_v
  - $\{u, v\} \in E$ iff P_u, P_v share a grid edge



Definition of EPG-Graphs

- EPG (for Edge intersection graphs of Paths on a Grid) graphs introduced in [GLS09]
- In EPG-graph $G = (V, E)$:
  - each vertex v corresponds to a path P_v
  - $\{u, v\} \in E$ iff P_u, P_v share a grid edge



- B_k-EPG: graphs having a representation where every path has at most k bends
- X-EPG $\subseteq B_1$-EPG (with $X \subseteq \{⌜, ⌝, ⌞, ⌟\}$): paths can only have shapes in X

Properties of EPG-Graphs

Class inclusions

- max deg $\Delta \subseteq B_\Delta$-EPG [HKU14]
- B_0-EPG $=$ interval graphs
- B_1-EPG $\subset$ 2-Track graphs $\subset$ B_3-EPG
- B_1-EPG $K_{3,3}$ induced free, $K_{3,3} - e$ induced free, $S_{n\geq 4}$ induced free [GLS09]

Properties of EPG-Graphs

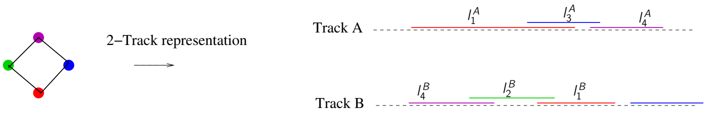
Class inclusions

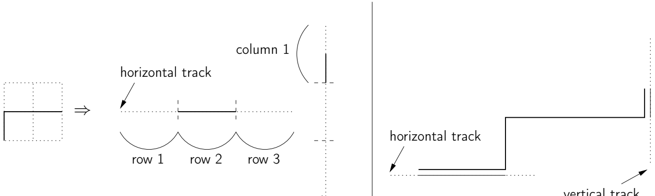
- max deg $\Delta \subseteq B_\Delta$-EPG [HKU14]
- B_0-EPG = interval graphs
- B_1-EPG $\subset$ 2-Track graphs $\subset$ B_3-EPG
- B_1-EPG $K_{3,3}$ induced free, $K_{3,3} - e$ induced free, $S_{n\geq 4}$ induced free [GLS09]

Properties of EPG-Graphs

Class inclusions

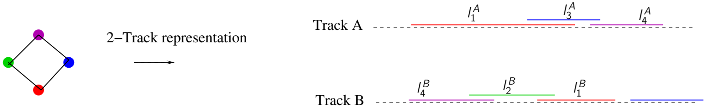
- max deg $\Delta \subseteq B_\Delta$-EPG [HKU14]
- B_0-EPG $=$ interval graphs
- B_1-EPG $\subset$ 2-Track graphs $\subset$ B_3-EPG
- B_1-EPG $K_{3,3}$ induced free, $K_{3,3} - e$ induced free, $S_{n\geq 4}$ induced free [GLS09]

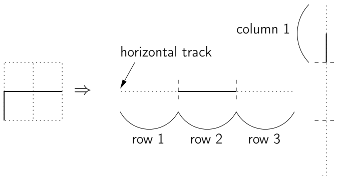


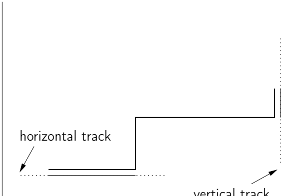


Class inclusions

- max deg $\Delta \subseteq B_\Delta$-EPG [HKU14]
- B_0-EPG $=$ interval graphs
- B_1-EPG $\subset$ 2-Track graphs $\subset$ B_3-EPG
- B_1-EPG $K_{3,3}$ induced free, $K_{3,3} - e$ induced free, $S_{n \geq 4}$ induced free [GLS09]







Properties of EPG-Graphs

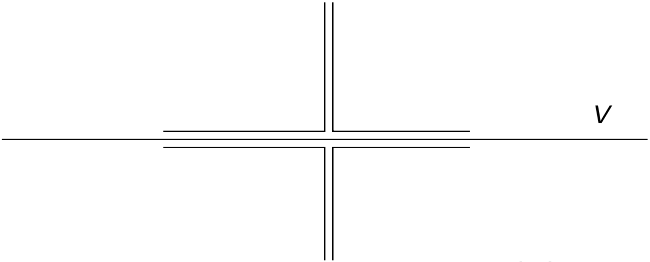
Bad news about B_1-EPG

- B_1-EPG are not planar graphs ($K_n \in B_1$-EPG)
- B_1-EPG are not perfect graphs ($C_n \in B_1$-EPG) (but $G[N(v)]$ weakly chordal)
- B_1-EPG do not benefit from the many results on "intersection graph class" as two paths can cross without creating an edge

Properties of EPG-Graphs

Bad news about B_1-EPG

- B_1-EPG are not planar graphs ($K_n \in B_1$-EPG)
- B_1-EPG are not perfect graphs ($C_n \in B_1$-EPG) (but $G[N(v)]$ weakly chordal)
- B_1-EPG do not benefit from the many results on "intersection graph class" as two paths can cross without creating an edge

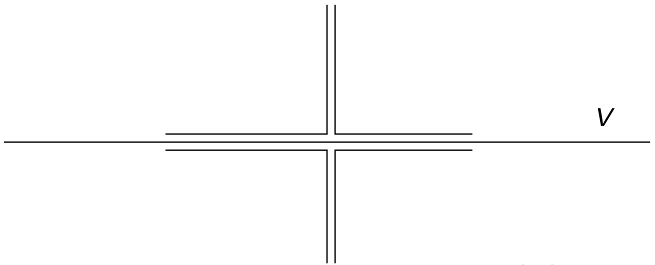


$N(v)$ is a C_4

Properties of EPG-Graphs

Bad news about B_1-EPG

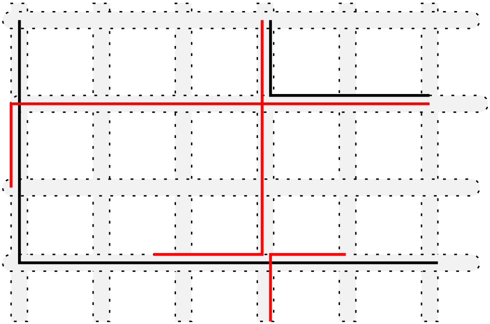
- B_1-EPG are not planar graphs ($K_n \in B_1$-EPG)
- B_1-EPG are not perfect graphs ($C_n \in B_1$-EPG) (but $G[N(v)]$ weakly chordal)
- B_1-EPG do not benefit from the many results on "intersection graph class" as two paths can cross without creating an edge



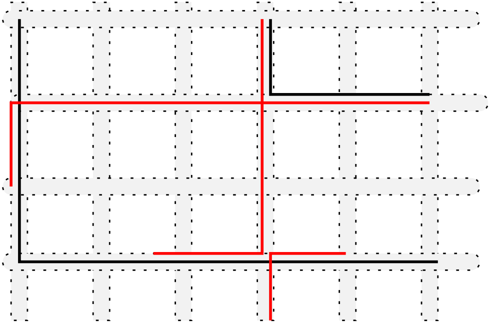
$N(v)$ is a C_4

Motivation

- Considered problem: MIS (Maximum Independent Set) on B_1-EPG graphs (supposing a representation is given)
- In 2013, [EGM13] proved that MIS (and coloring) are NP-hard on B_1-EPG and admits a 4-approximation algorithm
- Can we say more ? (approximability and fixed parameter tractability with standard parameterization)

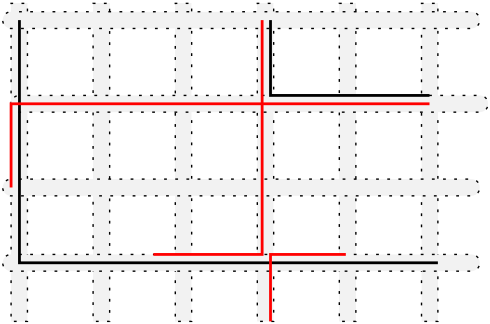


- Considered problem: MIS (Maximum Independent Set) on B_1-EPG graphs (supposing a representation is given)
- In 2013, [EGM13] proved that MIS (and coloring) are NP-hard on B_1-EPG and admits a 4-approximation algorithm
- Can we say more ? (approximability and fixed parameter tractability with standard parameterization)



Motivation

- Considered problem: MIS (Maximum Independent Set) on B_1-EPG graphs (supposing a representation is given)
- In 2013, [EGM13] proved that MIS (and coloring) are NP-hard on B_1-EPG and admits a 4-approximation algorithm
- Can we say more ? (approximability and fixed parameter tractability with standard parameterization)



Outline

Outline approximability

Related work

- simple 4 approximation on B_1-EPG ([EGM13] or [BYHN$^+$06])
- no PTAS for MIS on 2-Track as max deg $\Delta = 3 \subseteq$ 2-Track (recall B_1-EPG $\subset$ 2-Track)

Outline approximability

Related work

- simple 4 approximation on B_1-EPG ([EGM13] or [BYHN$^+$06])
- no PTAS for MIS on 2-Track as max deg $\Delta = 3 \subseteq$ 2-Track (recall B_1-EPG $\subset$ 2-Track)

Outline approximability

Related work

- simple 4 approximation on B_1-EPG ([EGM13] or [BYHN$^+$06])
- no PTAS for MIS on 2-Track as max deg $\Delta = 3 \subseteq$ 2-Track (recall B_1-EPG $\subset$ 2-Track)

Our contributions

- no PTAS for MIS on $\{\ulcorner\}$-EPG, even if each path has its vertical part or its horizontal part of length at most 3
- PTAS for MIS on B_1-EPG when each path has its horizontal part at most c

Negative result

Lemma: (no PTAS for $\{\ulcorner, \urcorner\}$-EPG)

There is a strict reduction from MAX-3-SAT(3) to $\{\ulcorner, \urcorner\}$-EPG

Proof

Consider first the textbook reduction from MAX-3-SAT to MIS as :

- create one triangle for each clause
- add an edge between any occurrence of literal and its negation
- assignment satisfying t clauses $\Leftrightarrow$ IS size t

Negative result

Lemma: (no PTAS for $\{\ulcorner, \urcorner\}$-EPG)

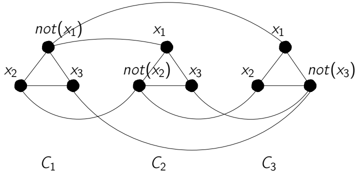
There is a strict reduction from MAX-3-SAT(3) to $\{\ulcorner, \urcorner\}$-EPG

Proof

Consider first the textbook reduction from MAX-3-SAT to MIS as :

- create one triangle for each clause
- add an edge between any occurrence of literal and its negation
- assignment satisfying t clauses $\Leftrightarrow$ IS size t

$C_1 = not(x_1) \vee x_2 \vee x_3$
$C_2 = x_1 \vee not(x_2) \vee x_3$
$C_3 = x_1 \vee x_2 \vee not(x_3)$



Negative result

Lemma: (no PTAS for $\{\ulcorner, \urcorner\}$-EPG)

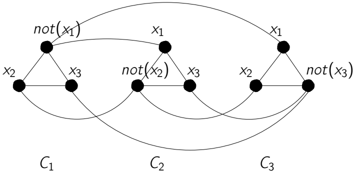
There is a strict reduction from MAX-3-SAT(3) to $\{\ulcorner, \urcorner\}$-EPG

Proof

Consider first the textbook reduction from MAX-3-SAT to MIS as :

- create one triangle for each clause
- add an edge between any occurrence of literal and its negation
- assignment satisfying t clauses $\Leftrightarrow$ IS size t

$C_1 = not(x_1) \vee x_2 \vee x_3$
$C_2 = x_1 \vee not(x_2) \vee x_3$
$C_3 = x_1 \vee x_2 \vee not(x_3)$



Negative result

Lemma: (no PTAS for $\{\ulcorner, \urcorner\}$-EPG)

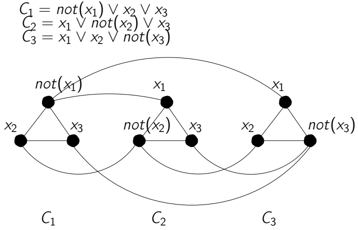
There is a strict reduction from MAX-3-SAT(3) to $\{\ulcorner, \urcorner\}$-EPG

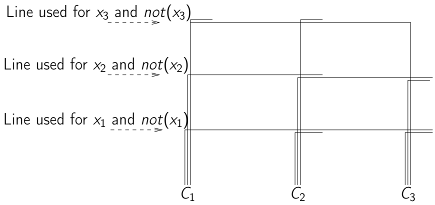
Proof

Consider first the textbook reduction from MAX-3-SAT to MIS as :

- create one triangle for each clause
- add an edge between any occurrence of literal and its negation
- assignment satisfying t clauses $\Leftrightarrow$ IS size t

As we reduce from MAX-3-SAT(3), the graph is $\{\ulcorner, \urcorner\}$-EPG

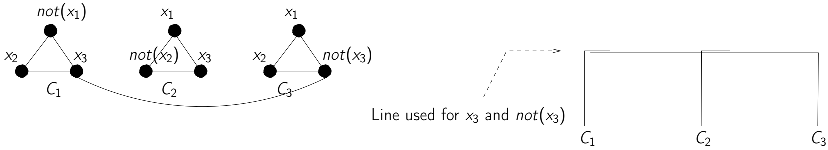




Negative result

How removing one type of shape ?

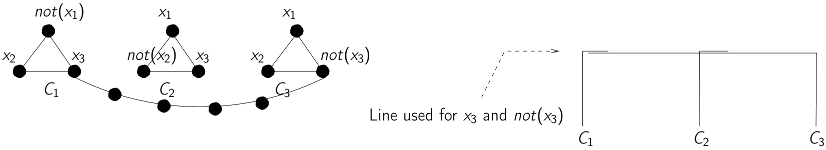
- Given the previous G, subdivide 4 times each "literral" edge to get a new graph G'
- G' can now be drawn as $\{\ulcorner\}$-EPG
- $\exists$ indep. set. S in $G \Leftrightarrow \exists$ indep. set. S' in G' with $|S'| = |S| + 2m$
- As $\exists c$ such that $m \leq cOpt(G)$, we get an AP-reduction for MIS from $\{\ulcorner, \urcorner\}$-EPG to $\{\ulcorner\}$-EPG



Negative result

How removing one type of shape ?

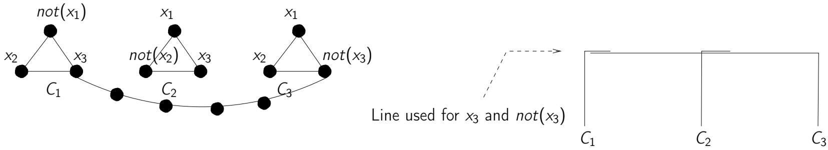
- Given the previous G, subdivide 4 times each "literral" edge to get a new graph G'
- G' can now be drawn as {⌜}-EPG
- $\exists$ indep. set. S in $G \Leftrightarrow \exists$ indep. set. S' in G' with $|S'| = |S| + 2m$
- As $\exists c$ such that $m \leq cOpt(G)$, we get an AP-reduction for MIS from {⌜, ⌝}-EPG to {⌜}-EPG



Negative result

How removing one type of shape ?

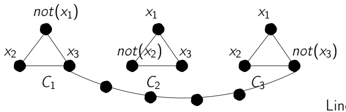
- Given the previous G, subdivide 4 times each "literral" edge to get a new graph G'
- G' can now be drawn as $\{\ulcorner\}$-EPG
- $\exists$ indep. set. S in $G \Leftrightarrow \exists$ indep. set. S' in G' with $|S'| = |S| + 2m$
- As $\exists c$ such that $m \leq cOpt(G)$, we get an AP-reduction for MIS from $\{\ulcorner, \urcorner\}$-EPG to $\{\ulcorner\}$-EPG

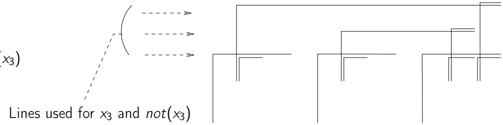


Negative result

How removing one type of shape ?

- Given the previous G, subdivide 4 times each "literral" edge to get a new graph G'
- G' can now be drawn as $\{\ulcorner\}$-EPG
- $\exists$ indep. set. S in $G \Leftrightarrow \exists$ indep. set. S' in G' with $|S'| = |S| + 2m$
- As $\exists c$ such that $m \leq cOpt(G)$, we get an AP-reduction for MIS from $\{\ulcorner, \urcorner\}$-EPG to $\{\ulcorner\}$-EPG

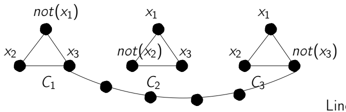


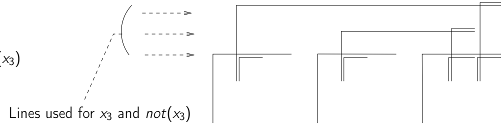


Negative result

How removing one type of shape ?

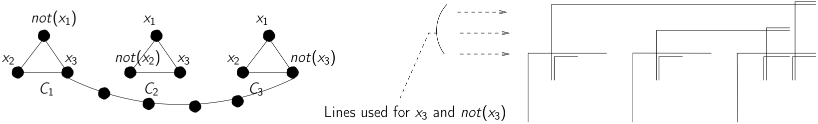
- Given the previous G, subdivide 4 times each "literral" edge to get a new graph G'
- G' can now be drawn as $\{\ulcorner\}$-EPG
- $\exists$ indep. set. S in $G \Leftrightarrow \exists$ indep. set. S' in G' with $|S'| = |S| + 2m$
- As $\exists c$ such that $m \leq cOpt(G)$, we get an AP-reduction for MIS from $\{\ulcorner, \urcorner\}$-EPG to $\{\ulcorner\}$-EPG





How removing one type of shape ?

- Given the previous G, subdivide 4 times each "literral" edge to get a new graph G'
- G' can now be drawn as $\{\ulcorner\}$-EPG
- $\exists$ indep. set. S in $G \Leftrightarrow \exists$ indep. set. S' in G' with $|S'| = |S| + 2m$
- As $\exists c$ such that $m \leq cOpt(G)$, we get an AP-reduction for MIS from $\{\ulcorner, \urcorner\}$-EPG to $\{\ulcorner\}$-EPG



Theorem

There is no PTAS for MIS on $\{\ulcorner\}$-EPG, even if each path has its vertical part or its horizontal part of length at most 3

Negative result

Can we improve this when both parts have constant size ? No.

Theorem

There is PTAS for MIS on B_1-EPG when each path has its horizontal part at most c (which remains NP-hard)

Proof

We will prove this using classical Baker shifting technique

Positive result

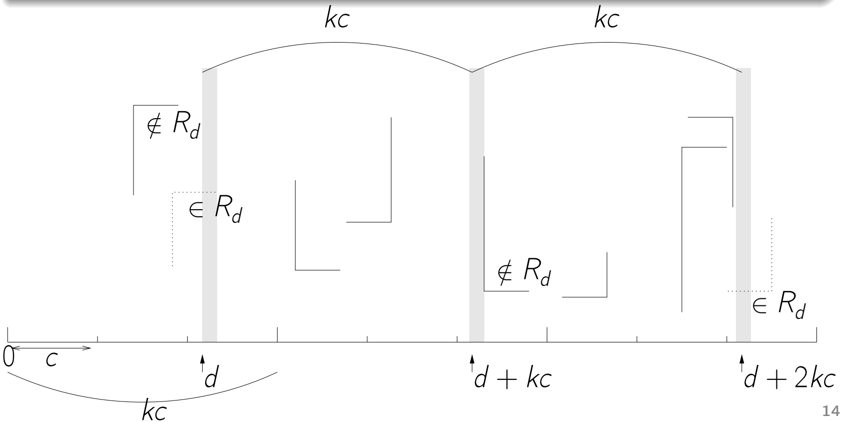
Proof

- Let k be a large integer (goal: sol of size $|S| \geq |OPT|(1 - \frac{1}{k})$)
- Given $d \in [kc - 1]$, let R_d be the set of paths whose horizontal part crosses the vertical strip d, $d + kc$, $d + 2kc$, ..
- Let $OPT_d = OPT \cap R_d$

Positive result

Proof

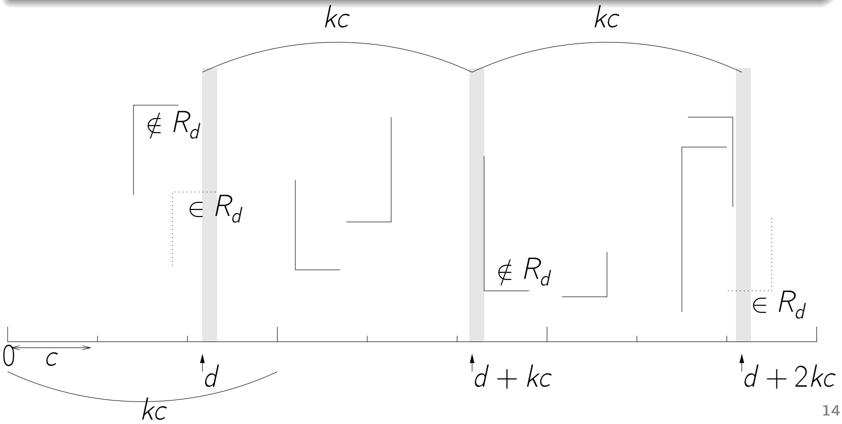
- Let k be a large integer (goal: sol of size $|S| \geq |OPT|(1 - \frac{1}{k})$)
- Given $d \in [kc - 1]$, let R_d be the set of paths whose horizontal part crosses the vertical strip d, $d + kc$, $d + 2kc$, ..
- Let $OPT_d = OPT \cap R_d$



Positive result

Proof

- Let k be a large integer (goal: sol of size $|S| \geq |OPT|(1 - \frac{1}{k})$)
- Given $d \in [kc - 1]$, let R_d be the set of paths whose horizontal part crosses the vertical strip d, $d + kc$, $d + 2kc$, ..
- Let $OPT_d = OPT \cap R_d$



Positive result

Lemma

$\exists d_0$ such that $|OPT_{d_0}| \leq \frac{1}{k}|OPT|$

Proof

- $\sum_{d=0}^{kc-1} |OPT_d| \leq c|OPT|$ (as each vertex belongs to at most c different R_d)
- as $|OPT_{d_0}|kc \leq \sum_{d=0}^{kc-1} |OPT_d|$, we get the result

Positive result

Proof

Back to the PTAS proof:

- thus, for each d we solve the problem optimally on $G \setminus R_d$, and we output A: the best of these solutions
- Previous Lemma $\Rightarrow |A| \geq (1 - \frac{1}{k})|OPT|$

Positive result

Proof

Back to the PTAS proof:

- thus, for each d we solve the problem optimally on $G \setminus R_d$, and we output A: the best of these solutions
- Previous Lemma $\Rightarrow |A| \geq (1 - \frac{1}{k})|OPT|$

Positive result

Proof

It remains to solve the problem optimally on $G \setminus R_d$:

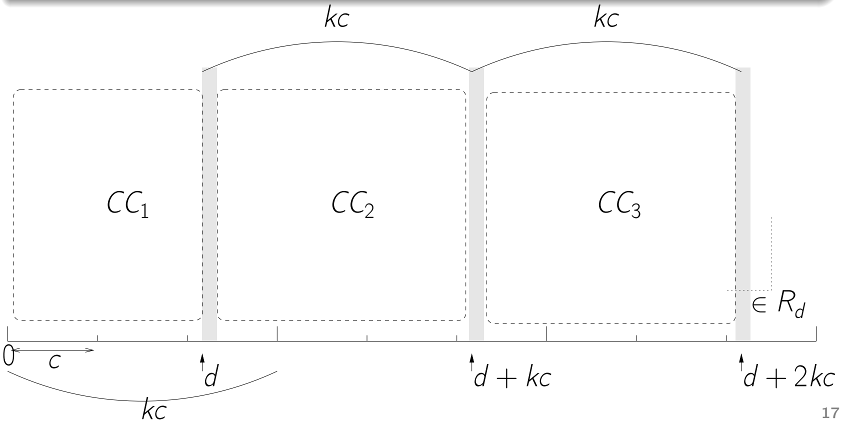
- $G \setminus R_d$ has several connected component CC_l, where each component is drawn on a grid of constant width kc
- we solve optimally on each CC_l using a dyn. prog. algorithm

Positive result

Proof

It remains to solve the problem optimally on $G \setminus R_d$:

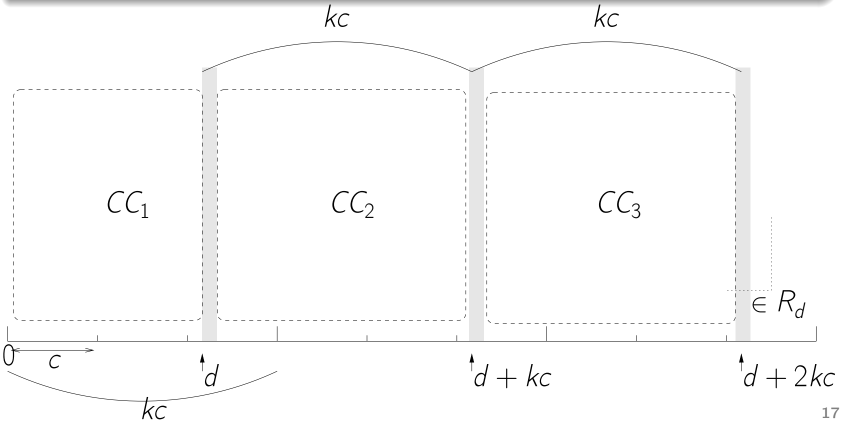
- $G \setminus R_d$ has several connected component CC_l, where each component is drawn on a grid of constant width kc
- we solve optimally on each CC_l using a dyn. prog. algorithm



Proof

It remains to solve the problem optimally on $G \setminus R_d$:

- $G \setminus R_d$ has several connected component CC_l, where each component is drawn on a grid of constant width kc
- we solve optimally on each CC_l using a dyn. prog. algorithm



Outline

Outline fixed parameter tractability

We consider the problem $OPT \leq k$? parameterized by k.
Question: is this problem FPT ? (*i.e.* can be solved in $f(k)poly(n)$)

Outline fixed parameter tractability

We consider the problem $OPT \leq k$? parameterized by k.
Question: is this problem FPT ? (*i.e.* can be solved in $f(k)poly(n)$)

Related work

B_1-EPG $\subset$ 2-Track $\subset$ B_3-EPG + MIS is W_1-hard on unit 2-Track [Jia10] $\Rightarrow$ MIS is W_1-hard on B_3-EPG

Outline fixed parameter tractability

We consider the problem $OPT \leq k$? parameterized by k.
Question: is this problem FPT ? (*i.e.* can be solved in $f(k)poly(n)$)

Related work

B_1-EPG $\subset$ 2-Track $\subset$ B_3-EPG + MIS is W_1-hard on unit 2-Track [Jia10] $\Rightarrow$ MIS is W_1-hard on B_3-EPG

Our contributions

- MIS is FPT on $\{\llcorner, \lrcorner, \ulcorner\}$-EPG
- MIS is W_1-hard on B_2-EPG

We consider the problem $OPT \leq k$? parameterized by k.
Question: is this problem FPT ? (*i.e.* can be solved in $f(k)poly(n)$)

Related work

B_1-EPG $\subset$ 2-Track $\subset$ B_3-EPG + MIS is W_1-hard on unit 2-Track [Jia10] $\Rightarrow$ MIS is W_1-hard on B_3-EPG

Our contributions

- MIS is FPT on $\{\llcorner, \lrcorner, \ulcorner\}$-EPG
- MIS is W_1-hard on B_2-EPG

We will prove that MIS is FPT on $\{\llcorner\}$-EPG using a branching algorithm.

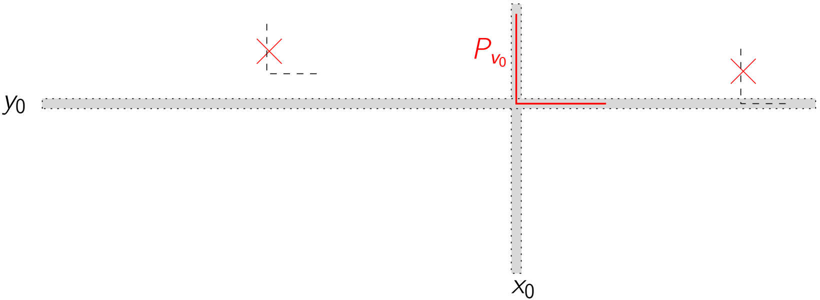
Definition

Given a path P, let $cor(P) = (x, y)$ be the coordinates of the corner of P in the grid (orienting as usual $\uparrow_y \rightarrow_x$)

MIS is FPT on {∟}-EPG

Proof

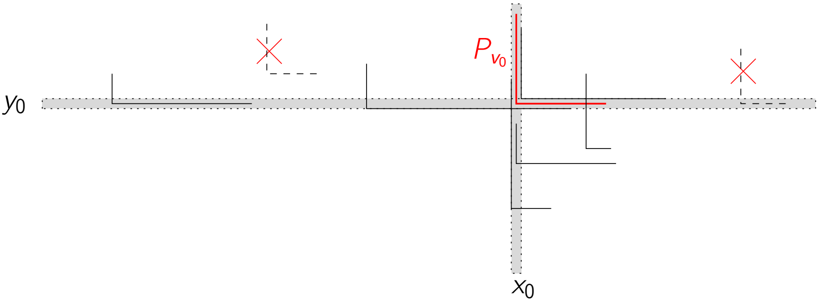
- Let P_{v_0} $(cor(P_{v_0}) = (x_0, y_0))$: a path whose corner is "top-right most" (highest line of the grid, and then the right most)
- Suppose $|OPT| = k$
- We know that $\exists P_{v^*} \in N[P_{v_0}] \cap OPT$ $(cor(P_{v^*}) = (x^*, y^*))$
- Let us find this P_{v^*} (or a $P' \in OPT'$) using at most $f(k)$ branches



MIS is FPT on {⌞}-EPG

Proof

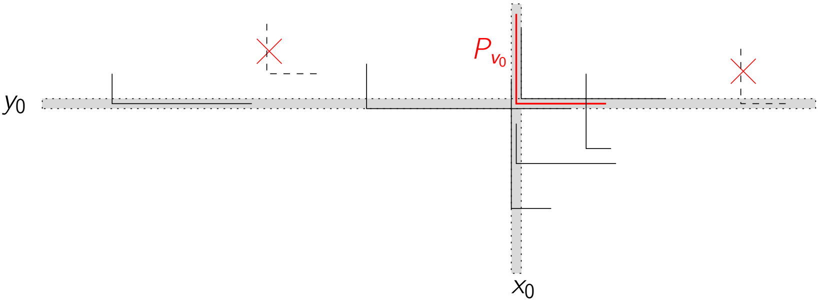
- Let P_{v_0} ($cor(P_{v_0}) = (x_0, y_0)$): a path whose corner is "top-right most" (highest line of the grid, and then the right most)
- Suppose $|OPT| = k$
- We know that $\exists P_{v^*} \in N[P_{v_0}] \cap OPT$ ($cor(P_{v^*}) = (x^*, y^*)$)
- Let us find this P_{v^*} (or a $P' \in OPT'$) using at most $f(k)$ branches



MIS is FPT on {⌞}-EPG

Proof

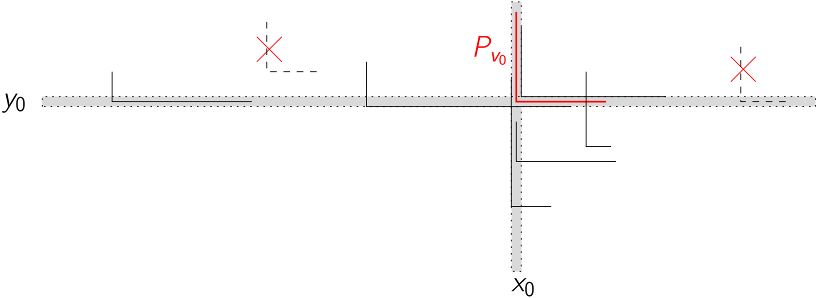
- Let P_{v_0} ($cor(P_{v_0}) = (x_0, y_0)$): a path whose corner is "top-right most" (highest line of the grid, and then the right most)
- Suppose $|OPT| = k$
- We know that $\exists P_{v^*} \in N[P_{v_0}] \cap OPT$ ($cor(P_{v^*}) = (x^*, y^*)$)
- Let us find this P_{v^*} (or a $P' \in OPT'$) using at most $f(k)$ branches



MIS is FPT on $\{\llcorner\}$-EPG

Proof

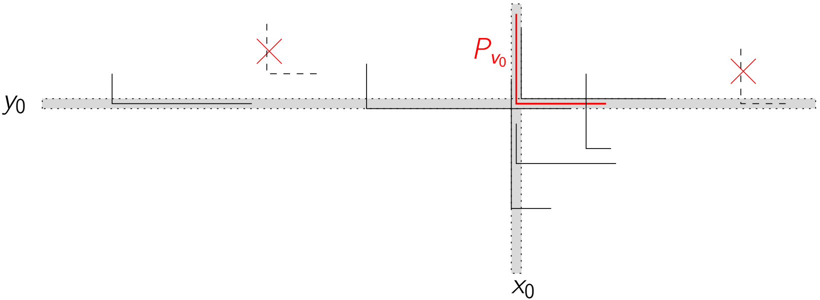
- Let P_{v_0} ($cor(P_{v_0}) = (x_0, y_0)$): a path whose corner is "top-right most" (highest line of the grid, and then the right most)
- Suppose $|OPT| = k$
- We know that $\exists P_{v^*} \in N[P_{v_0}] \cap OPT$ ($cor(P_{v^*}) = (x^*, y^*)$)
- Let us find this P_{v^*} (or a $P' \in OPT'$) using at most $f(k)$ branches



MIS is FPT on {⌞}-EPG

Proof

- Let P_{v_0} $(cor(P_{v_0}) = (x_0, y_0))$: a path whose corner is "top-right most" (highest line of the grid, and then the right most)
- Suppose $|OPT| = k$
- We know that $\exists P_{v^*} \in N[P_{v_0}] \cap OPT$ $(cor(P_{v^*}) = (x^*, y^*))$
- Let us find this P_{v^*} (or a $P' \in OPT'$) using at most $f(k)$ branches

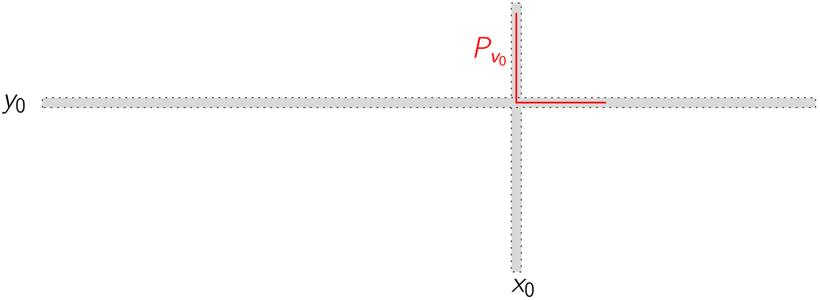


MIS is FPT on {└}-EPG

Proof

We branch on the following cases:

1. $x^* = x_0$ and $y^* = y_0$: easy, take P_{v_0}
2. $x^* = x_0$ and $y^* < y_0$
3. $x^* < x_0$ and $y^* = y_0$

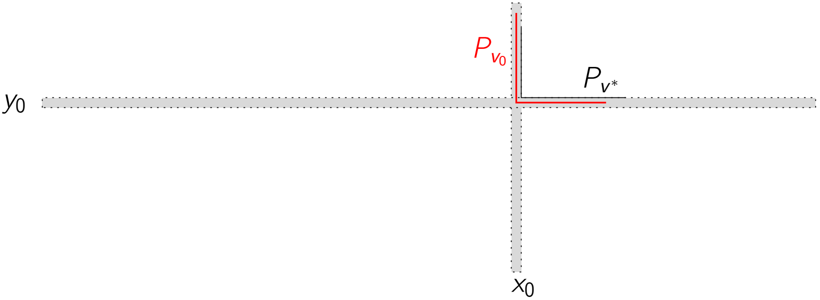


MIS is FPT on $\{\llcorner\}$-EPG

Proof

We branch on the following cases:

1. $x^* = x_0$ and $y^* = y_0$: easy, take P_{v_0}
2. $x^* = x_0$ and $y^* < y_0$
3. $x^* < x_0$ and $y^* = y_0$

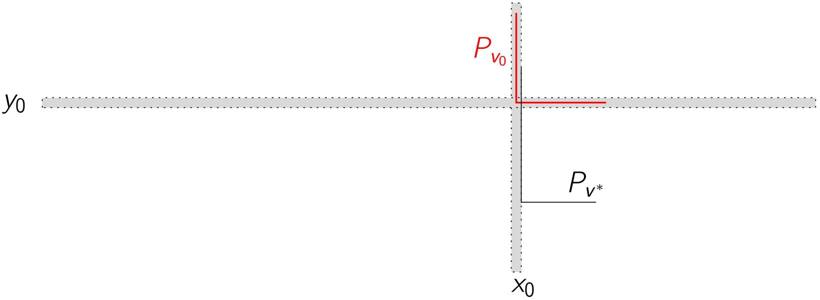


MIS is FPT on {⌞}-EPG

Proof

We branch on the following cases:

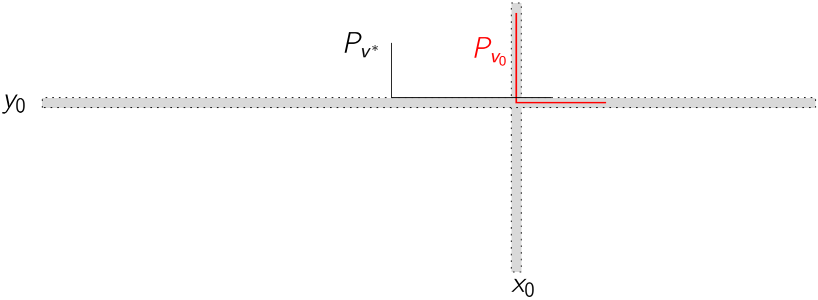
1. $x^* = x_0$ and $y^* = y_0$: easy, take P_{v_0}
2. $x^* = x_0$ and $y^* < y_0$
3. $x^* < x_0$ and $y^* = y_0$



Proof

We branch on the following cases:

1. $x^* = x_0$ and $y^* = y_0$: easy, take P_{v_0}
2. $x^* = x_0$ and $y^* < y_0$
3. $x^* < x_0$ and $y^* = y_0$



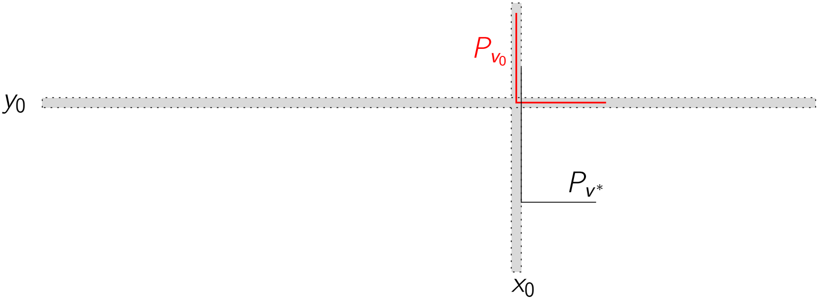
MIS is FPT on $\{\llcorner\}$-EPG

Proof

We branch on the following cases:

1. $x^* = x_0$ and $y^* = y_0$: easy, take P_{v_0}
2. $x^* = x_0$ and $y^* < y_0$
3. $x^* < x_0$ and $y^* = y_0$

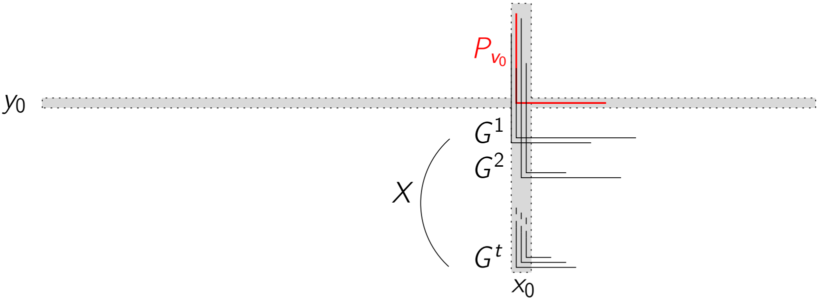
Let us only treat case 2 here (case 3 works similarly)



MIS is FPT on $\{\llcorner\}$-EPG

Proof

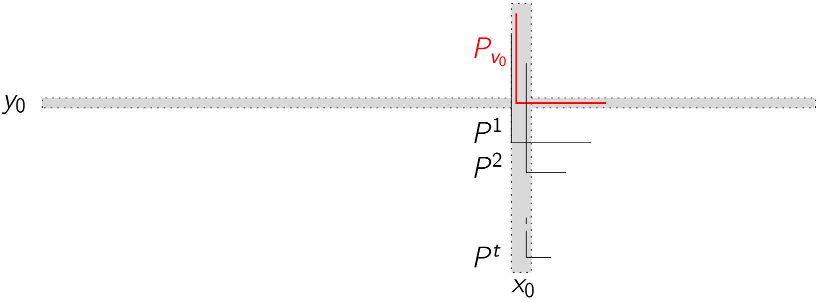
- Let X be the "vertical" neighbors of P_{v_0} ($P_{v^*} \in X$).
- Let partition $X = G^1 \cup G^2 .. \cup G^t$
- Sufficient to keep the left most path P^i in each G^i



MIS is FPT on $\{\llcorner\}$-EPG

Proof

- Let X be the "vertical" neighbors of P_{v_0} ($P_{v^*} \in X$).
- Let partition $X = G^1 \cup G^2 .. \cup G^t$
- Sufficient to keep the left most path P^i in each G^i

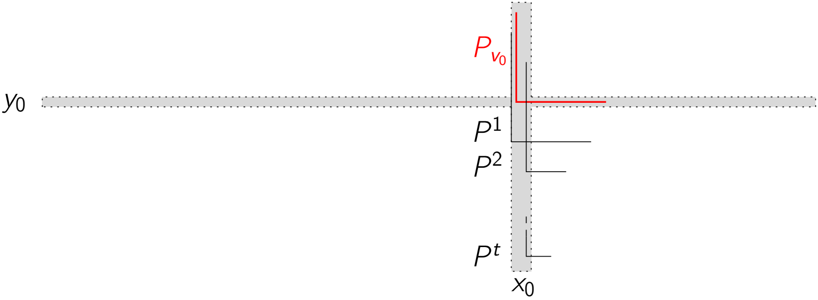


MIS is FPT on {∟}-EPG

Proof

Now, two cases are possible

1. $P_{v^*} \in \{P^1, \dots, P^k\}$: easy, guess which P^i to take
2. $P_{v^*} \in \{P^{k+1}, \dots, P^t\}$:
   - as $|OPT| = k$, there exists $P^i \in \{P^1, \dots, P^k\}$ such that there is no $P' \in OPT$ in the "horizontal line" of P^i
   - guess this P^i and take it

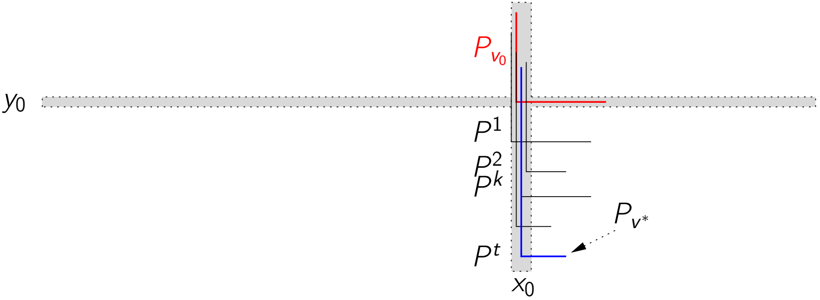


MIS is FPT on $\{\llcorner\}$-EPG

Proof

Now, two cases are possible

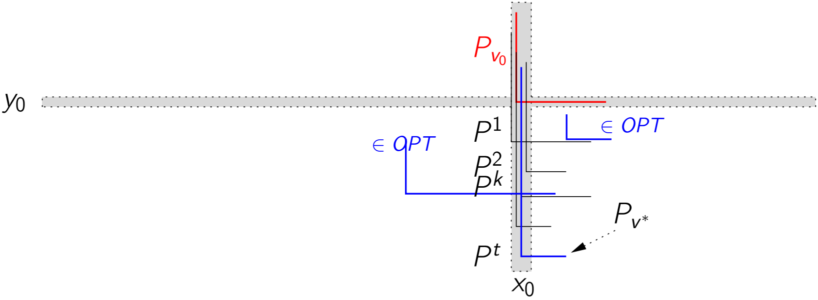
1. $P_{v^*} \in \{P^1, \ldots, P^k\}$: easy, guess which P^i to take
2. $P_{v^*} \in \{P^{k+1}, \ldots, P^t\}$:
   - as $|OPT| = k$, there exists $P^i \in \{P^1, \ldots, P^k\}$ such that there is no $P' \in OPT$ in the "horizontal line" of P^i
   - guess this P^i and take it



Proof

Now, two cases are possible

1. $P_{v^*} \in \{P^1, \ldots, P^k\}$: easy, guess which P^i to take
2. $P_{v^*} \in \{P^{k+1}, \ldots, P^t\}$:
   - as $|OPT| = k$, there exists $P^i \in \{P^1, \ldots, P^k\}$ such that there is no $P' \in OPT$ in the "horizontal line" of P^i
   - guess this P^i and take it

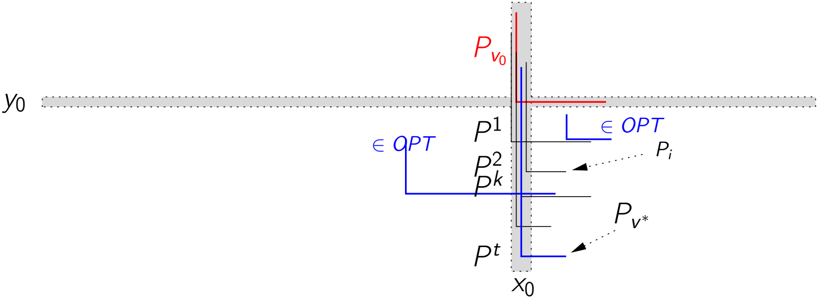


MIS is FPT on $\{\llcorner\}$-EPG

Proof

Now, two cases are possible

1. $P_{v^*} \in \{P^1, \ldots, P^k\}$: easy, guess which P^i to take
2. $P_{v^*} \in \{P^{k+1}, \ldots, P^t\}$:
   - as $|OPT| = k$, there exists $P^i \in \{P^1, \ldots, P^k\}$ such that there is no $P' \in OPT$ in the "horizontal line" of P^i
   - guess this P^i and take it



Conclusion on FPT

Positive results

- MIS is FPT on $\{\llcorner\}$-EPG as we can find a $P_{v^*} \in OPT$ using at most $f(k)$ branches
- The previous appproach doesn't work (even for 2 shapes)..
- but we use the same kind of arguments to prove that MIS is FPT with 3 shapes

Conclusion on FPT

Positive results

- MIS is FPT on $\{\llcorner\}$-EPG as we can find a $P_{v^*} \in OPT$ using at most $f(k)$ branches
- The previous appproach doesn't work (even for 2 shapes)..
- but we use the same kind of arguments to prove that MIS is FPT with 3 shapes

Conclusion on FPT

Positive results

- MIS is FPT on $\{\llcorner\}$-EPG as we can find a $P_{v^*} \in OPT$ using at most $f(k)$ branches
- The previous appproach doesn't work (even for 2 shapes)..
- but we use the same kind of arguments to prove that MIS is FPT with 3 shapes

Conclusion on FPT

Positive results

- MIS is FPT on $\{\llcorner\}$-EPG as we can find a $P_{v^*} \in OPT$ using at most $f(k)$ branches
- The previous appproach doesn't work (even for 2 shapes)..
- but we use the same kind of arguments to prove that MIS is FPT with 3 shapes

Negative results

- MIS is W_1-hard on B_2-EPG

Conclusion

Approximability

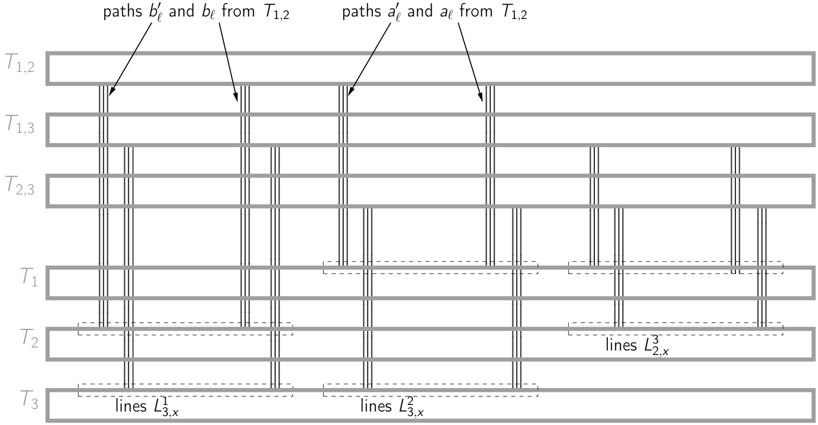
- No PTAS unless $P = NP$
- PTAS when one side is always small
- Simple 4 approximation
- Open: $c < 4$ approximation ?

Fixed parameter tractability

- FPT with 3 shapes
- W_1-hard on B_2-EPG
- Open: FPT on B_1-EPG ?

Thank you for your attention ! ...

MIS is W_1-hard on B_2-EPG as:



Bibliography

[BYHN+06] Reuven Bar-Yehuda, Magnús M Halldórsson, Joseph Seffi Naor, Hadas Shachnai, and Irina Shapira.
Scheduling split intervals.
SIAM J. Comput., 36(1):1–15, 2006.

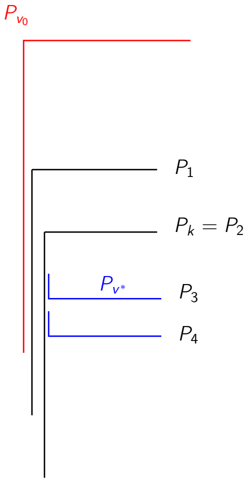
[EGM13] Dror Epstein, MartinCharles Golumbic, and Gila Morgenstern.
Approximation algorithms for b_1-epg graphs.
In WADS 2013: Algorithms and Data Structures, volume 8037 of Lecture Notes in Computer Science, pages 328–340. Springer Berlin Heidelberg, 2013.

[GLS09] Martin Charles Golumbic, Marina Lipshteyn, and Michal Stern.
Edge intersection graphs of single bend paths on a grid.
Networks, 54(3):130–138, 2009.

[HKU14] Daniel Heldt, Kolja Knauer, and Torsten Ueckerdt.
Edge-intersection graphs of grid paths: The bend-number.
Discrete Applied Mathematics, 167(0):144 – 162, 2014.

[Jia10] Minghui Jiang.
On the parameterized complexity of some optim ization problems related to multiple-interval graphs.
Theoretical Computer Science, 411:4253–4262, 2010.

Why does it fail for 2 shapes ?



$P_{v^*} \in \{P_{k+1}, \ldots, P_t\}$
but we cannot restructure OPT to chose one of the $\{P_1, \ldots, P_k\}$