Combining Multiple Heuristics on Discrete Resources

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1. Presentation of the problem

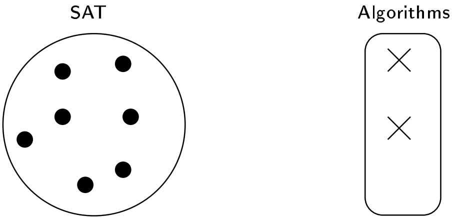
2. Oracle formalism for PTAS design

3. Approximation schemes for the restricted dRSSP
   - First guess : arbitrary subset
   - Second guess : convenient subset

Introduction

Context:

- a finite set H of algorithms/heuristics (algorithm portfolio)
- a finite set I of "representatives" instances (benchmark)
- the time needed by each algorithm of H to solve each instance of I is known
- goal : minimize the time needed to solve all the instances from the benchmark
- more than selection : combination of algorithms



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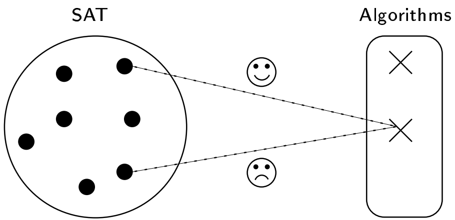

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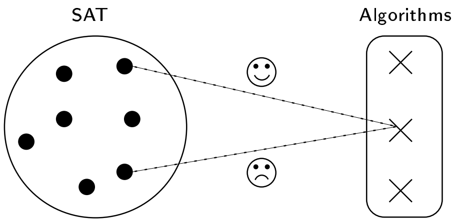

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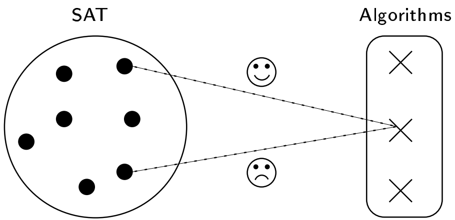

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What we mean by combination :

- one instance may be solved by several algorithms in parallel
- parallel task model : moldable
- when a solution of an instance is found, everyone is aware

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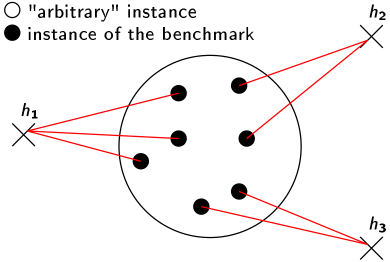

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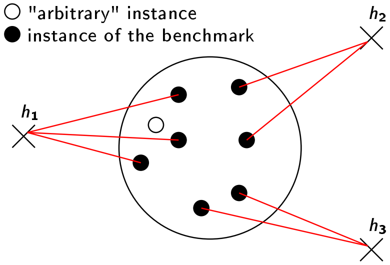

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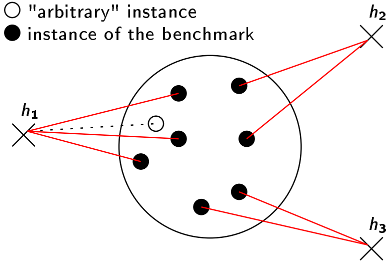

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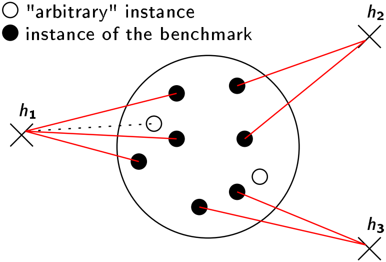

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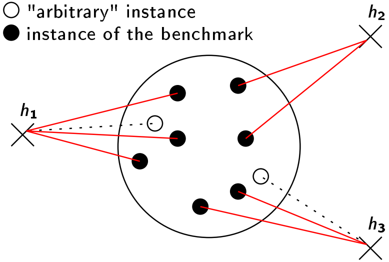

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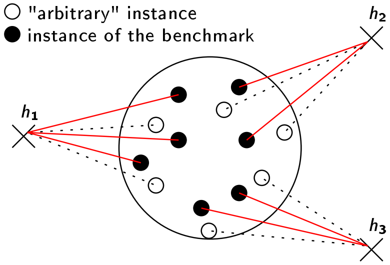

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Some existing combination models:

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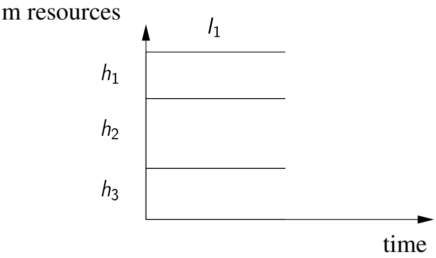

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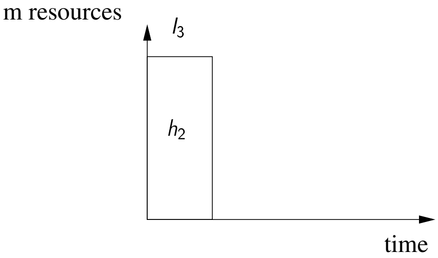

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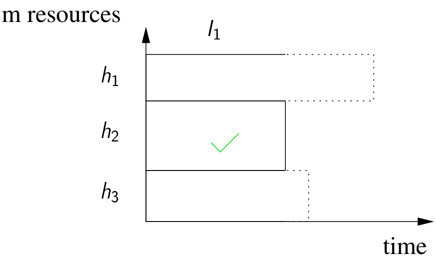

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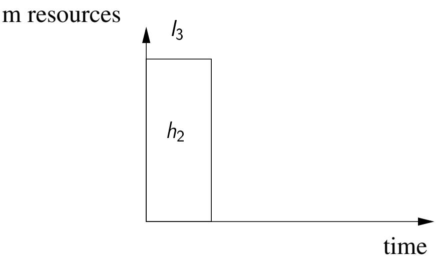

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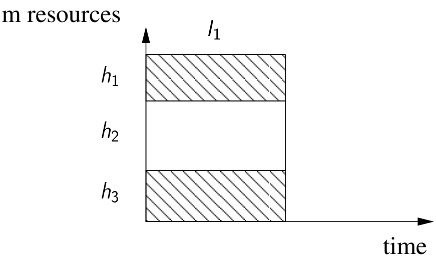

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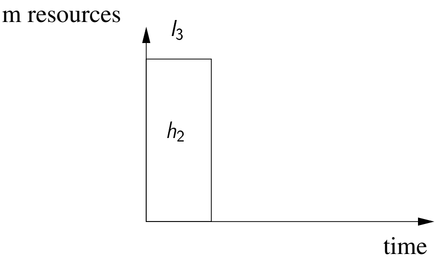

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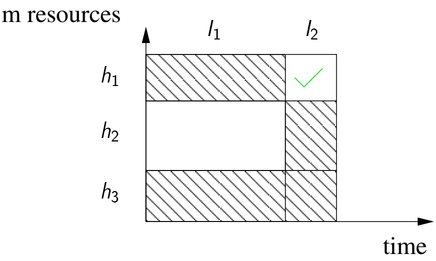

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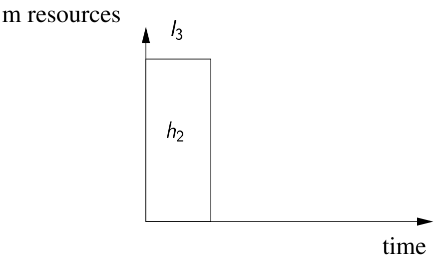

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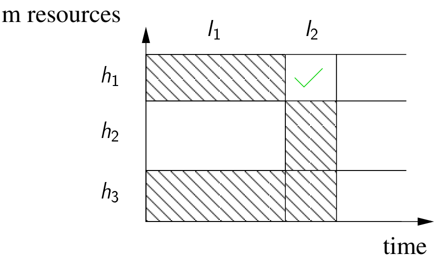

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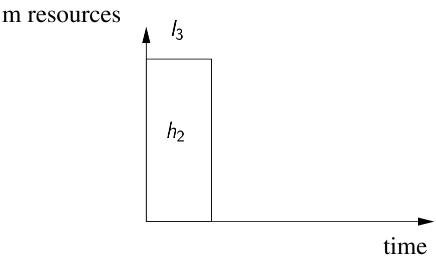

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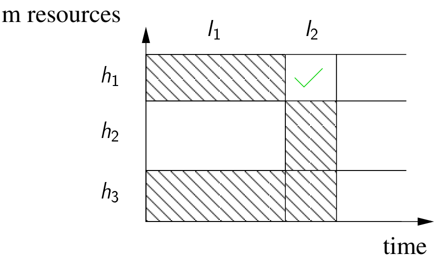

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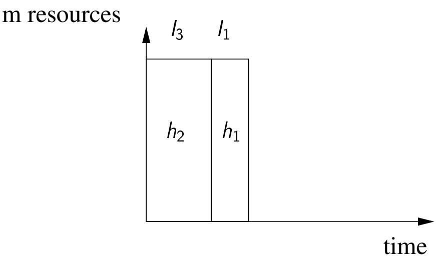

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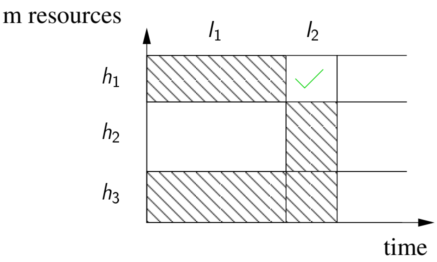

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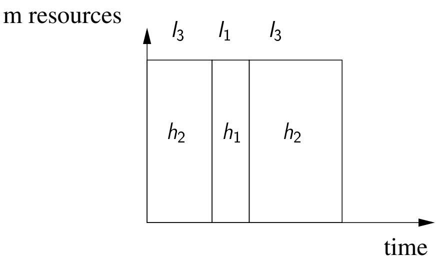

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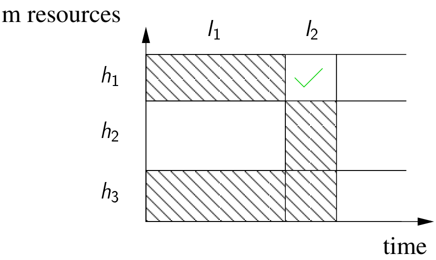

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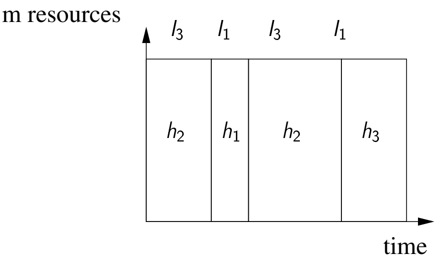

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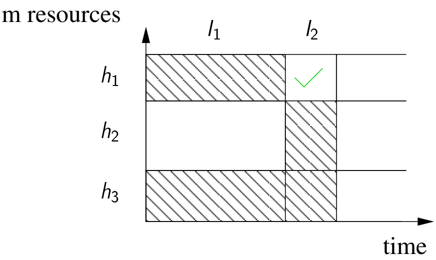

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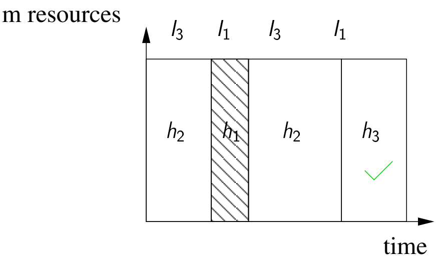

Definition of the dRSSP

Input of the discrete Resource Sharing Scheduling Problem:

- a finite set of instances $I = \{I_1, \ldots, I_n\}$
- a finite set of heuristics $H = \{h_1, \ldots, h_k\}$
- m identical resources
- a cost $C(h_i, I_j, p) \in R^+$ for each $I_j \in I$, $h_i \in H$ and $p \in \{1, \ldots, m\}$

Output : an allocation $S = (S_1, \ldots, S_k)$ such that:

- $S_i \in \mathbb{N}$
- $\sum_{i=1}^{k} S_i = m$
- S minimizes $\sum_{j=1}^{n} \min_{1 \le i \le k} \{C(h_i, I_j, S_i)\}$

Continuous version ($p \in R^+$) in [1].

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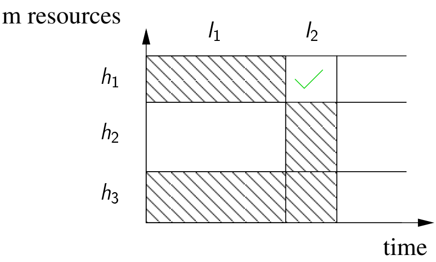
Complexity results

- the dRSSP is NP hard in the strong sense (reduction from the vertex cover problem)
- the dRSSP is innaproximable (unless $P = NP$) within a constant factor (if $m < k$)
- we study a restricted version:
  - **linear cost assumption:** $C(h_i, I_j, p) = C(h_i, I_j, m)\frac{m}{p}$
  - **well chosen portfolio:** each heuristic must use at least one processor ($S_i \geq 1$)

Notations

Given a solution S:

- let $\sigma(j) = \underset{1 \leq i \leq k}{argmin} \frac{C(h_i, I_j)}{S_i}$ be the index of the used heuristic for instance $j \in \{1, .., n\}$ in S
- let $T(I_j) = \frac{C(h_{\sigma(j)}, I_j)}{S_{\sigma(j)}}$ be the processing time of instance j in S
- let $T(h_i) = \Sigma_{j/\sigma(j)=i} T(I_j)$ is the "useful" computation time of heuristic i in S



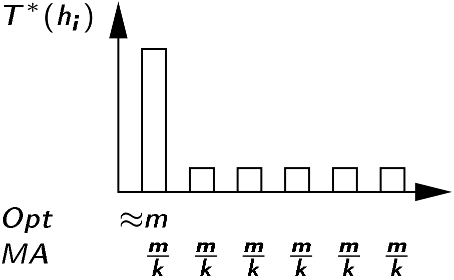
A naive algorithm : MA

We consider the mean-allocation (MA) algorithm which simply allocates $\lfloor \frac{m}{k} \rfloor$ resources to each heuristic.

Proposition

MA is a k approximation.

Proof/Worst case:



1. Presentation of the problem
2. Oracle formalism for PTAS design
3. Approximation schemes for the restricted dRSSP
   - First guess : arbitrary subset
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Introduction

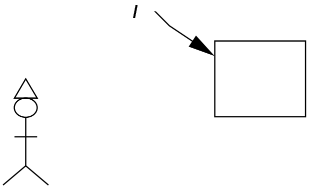
Some of the main *PTAS* design techniques [2]:

- structuring the input
- structuring the output ("extending partial small size solutions")
- structuring the execution of an algorithm ("trimmed algorithm")
- rounding *LP*
- oracle based approach
- ...

Oracle based approach

This vision is based on guesses from a reliable oracle. Given an instance I, the main ("polynomial") steps are:

- define the guess G: choose an "interesting" property P
- ask a question $Q(I)$ to the oracle
- the oracle provides an answer $r^* \in R$ (s t. $P(Q(I), r^*)$ is true)
- find a solution using the guess: an algorithm A provides $S(r^*) \leq \rho Opt$
- take the best: try all the possible answers and select the best of all the $S(r), r \in R$



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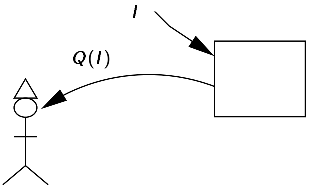

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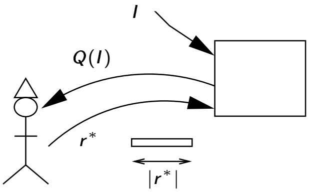

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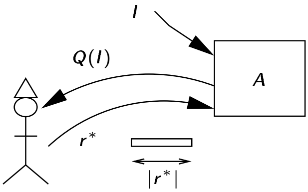

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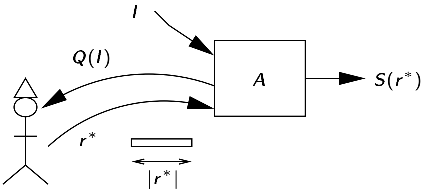

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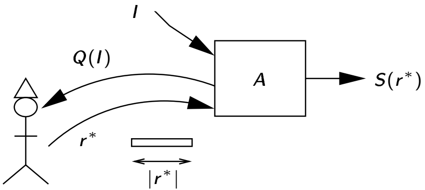

Oracle based approach

Thus, the obtained algorithm (without oracle):

- is a ρ approximation
- has a computational complexity in $O(t_A * 2^{|r^*|})$

Generally, we can choose $|r^*|$ (leading to different ρ), leading to classical approximation schemes.

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Introduction

In this part:

- we use the *MA* algorithm as a basis
- we look for “the right” question to ask to the oracle

Introduction

We consider the following MA^G algorithm (given any guess $G = (X_1, \ldots, X_g), X_i \geq 1)$:

- allocate X_i processors to heuristic $h_i, i \in \{1, \ldots, g\}$
- applies MA on the k' others heuristics with the m' remaining processors

We will use this algorithm with guesses from the oracle.

Guess 1

As a first step we choose arbitrarily g heuristics denoted by $\{h_1, \ldots, h_g\}$.

Definition G1

Let $G_1 = (S_1^*, \ldots, S_g^*)$, for a fixed subset of g heuristics and a fixed optimal solution S^*.

Notice that $|G_1| = glog(m)$.
We use the algorithm MA^G with $G = G_1$.

Proposition

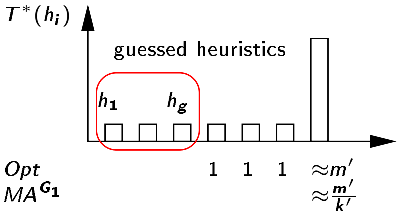
MA^{G_1} is a $k - g$ approximation.

Analysis of MA^{G_1}

We need some notations :

- let $k' = k - g$ be the number of remaining heuristics
- let $s = \Sigma_{i=1}^{g} S_i^*$ the number of processors used in the guess
- let $m' = m - s$ the number of remaining processors

Proof/Worst case:



Algorithm MA_R^G

The ratio for instances solved by the guessed heuristics is unnecessarily good.
Thus, the mean-allocation-reassign algorithm (MA_R^G)(given any guess $G = (X_1, \ldots, X_g), X_i \geq 1$):

- allocates only $X_i - \lfloor \frac{X_i}{\alpha} \rfloor$ processors to heuristic $h_i, i \in \{1, \ldots, g\}$
- applies MA on the k' others heuristics with the remaining processors

Proposition

With the "right" α, $MA_R^{G_1}$ is a $(k-g)(1-\beta)$ approximation

MA_R^G requires a larger guess to ensure that $s > k + c$ (c constant)

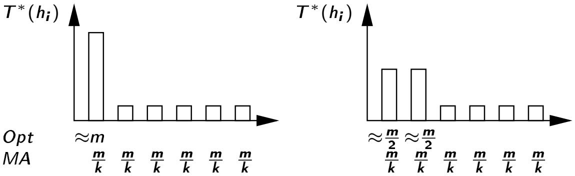
Introduction

The oracle response $r^* = [(i_1^*, \ldots, i_g^*), (r_1^*, \ldots, r_g^*)]$ will indicate the number of resources r_j^* allocated to heuritic $h_{i_j^*}$ (in a optimal solution).
The question now is: given an optimal solution S^*, what is the most "important" subset of g heuristics ?

- 1) those that have the largest number of allocated resources
- 2) those that have the fewest number of allocated resources
- 3) those that have the largest "useful" computation time

Another analysis of MA

For any heuristic $h_i, i \in \{1, .., k\}$, remember that $T^*(h_i) = \Sigma_{j/\sigma^*(j)=i} T^*(I_j)$ is the "useful" computation time of heuristic i in the solution S^*.



Difficult instances are the ones where the optimal only uses a small number of heuristics.

Another analysis of MA

$$
\begin{aligned}
T_{MA} &= \sum_{i=1}^{k} \sum_{j/\sigma^*(j)=i} T(I_j) \\
&\leq \sum_{i=1}^{k} \frac{S_i^*}{S_i} \sum_{j/\sigma^*(j)=i} T^*(I_j) \\
&= \sum_{i=1}^{k} \frac{S_i^*}{S_i} T^*(h_i) \\
&\leq Max_i(T^*(h_i)) \frac{m}{\lfloor \frac{m}{k} \rfloor} \\
&\leq Max_i(T^*(h_i))(2k-1)
\end{aligned}
$$

Guess 2

Definition

Let $G_2 = (S_1^*, \ldots, S_g^*)$, such that $T^*(h_1) \geq .. \geq T^*(h_g) \geq T^*(h_i), \forall i \in \{g+1, .., k\}$ in a fixed optimal solution S^*.

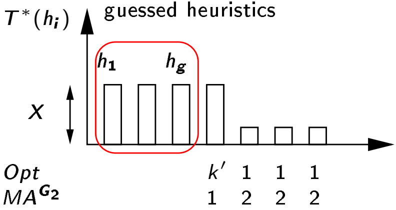
Notice that $|G_2| = glog(k) + glog(m)$.
We use the algorithm MA^G with $G = G_2$.

Proposition

MA^{G_2} is a $\frac{k}{g+1}$ approximation.

Analysis of MA^{G_2}

Proof/Worst case:



$$Opt \approx (g+1)X$$
$$MA^{G_2} \approx gX + k'X = kX$$

Outline of the obtained approximation schemes

algorithm	approx. ratio	$\|r^*\|$	complexity
MA^{G_1}	$(k-g)$	$glog(m)$	$O(m^g * kn)$
$MA_R^{G_1}$	$(k-g)(1-\beta)$	$log(k) + glog(m)$	$O(k * m^g * kn)$
MA^{G_2}	$\frac{k}{g+1}$	$g(log(k) + log(m))$	$O((km)^g * kn)$

Remark: these results are approximation schemes, and not $PTAS$

Conclusion

In this presentation:

- we extended the resource sharing problem to the discrete version (dRSSP)
- we presented the oracle methodology for PTAS design
- we built different approximation schemes for the restricted dRSSP

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Bibliography

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NP hardness

The reduction is from the vertex cover problem.
The input of the vertex cover problem is:

- k vertices
- n edges
- is there a vertex cover of size x ?

NP hardness

The input of the dRSSP is:

- k heuristics
- n instances in the benchmark
- x resources
- a cost matrix as following (costs are indicated when using every resources)

	I_1	I_2	I_3	..	I_n
h_1	..	..	T+1	..	..
h_2	..	..	α	..	..
..	..	..	T+1	..	..
..	..	..	T+1	..	..
h_k	..	..	α	..	..

How to choose the right value for T:

- if there exists a vertex cover of size x, then $opt \leq nx\alpha = T$
- else $opt \geq T + 1$

The gap can be arbitrary large.

The reduction for the restricted version is based on the same idea.