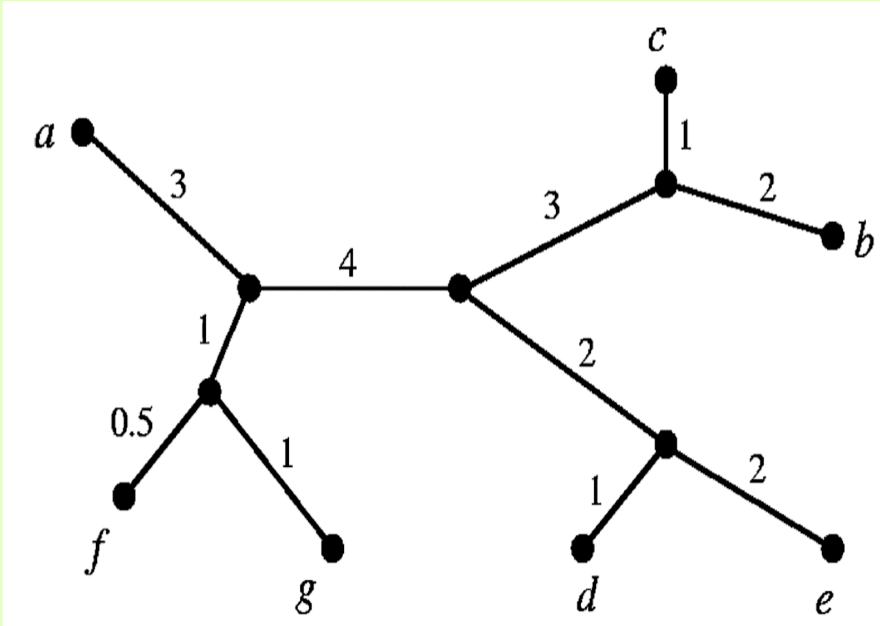
Encoding phylogenetic trees in terms of weighted quartets



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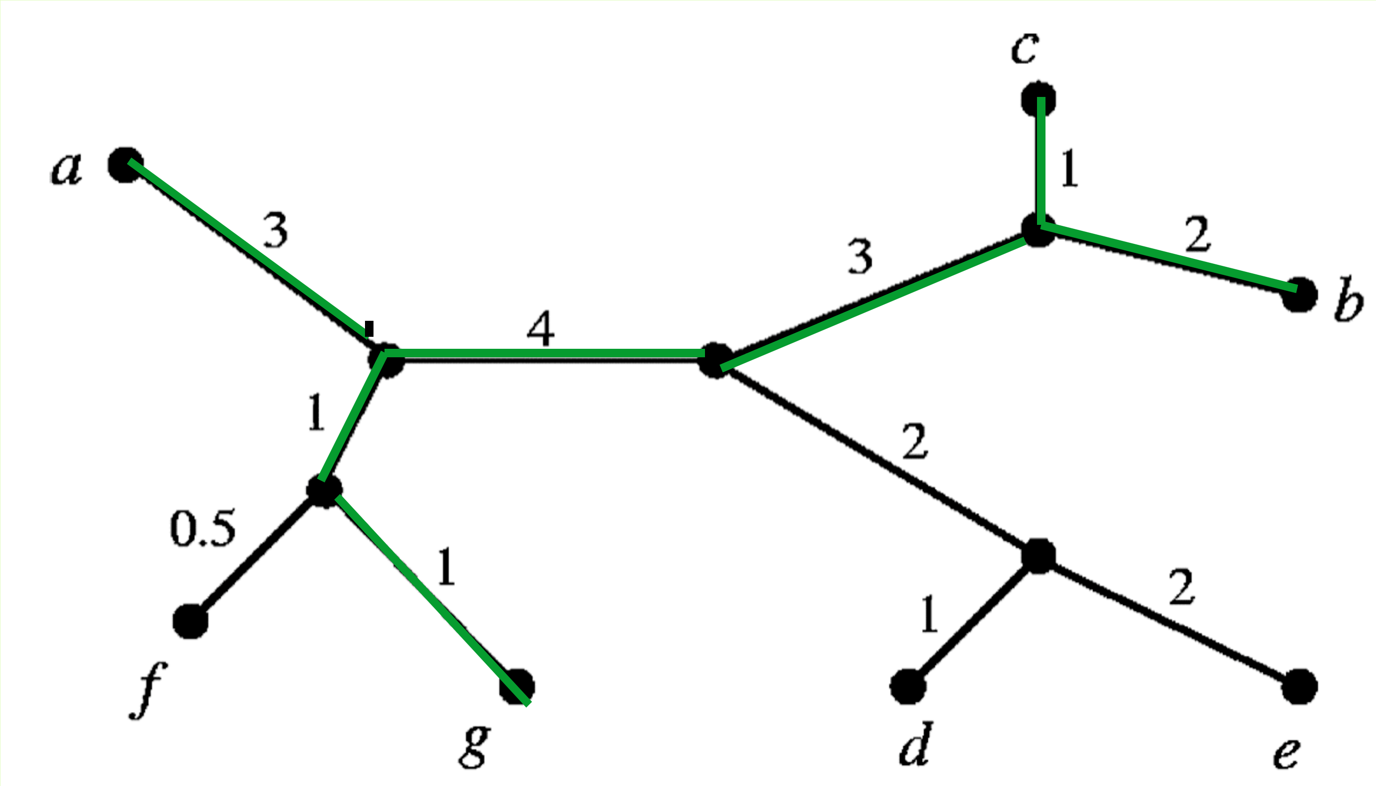
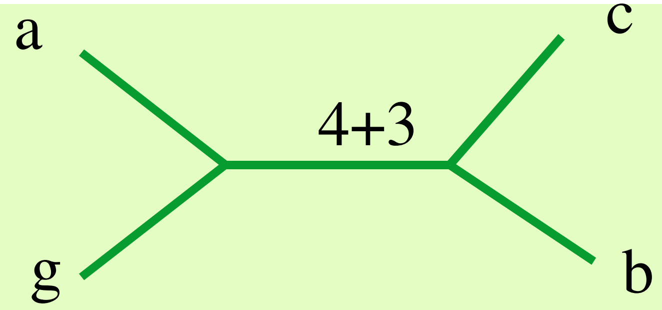
Weighted quartets from trees

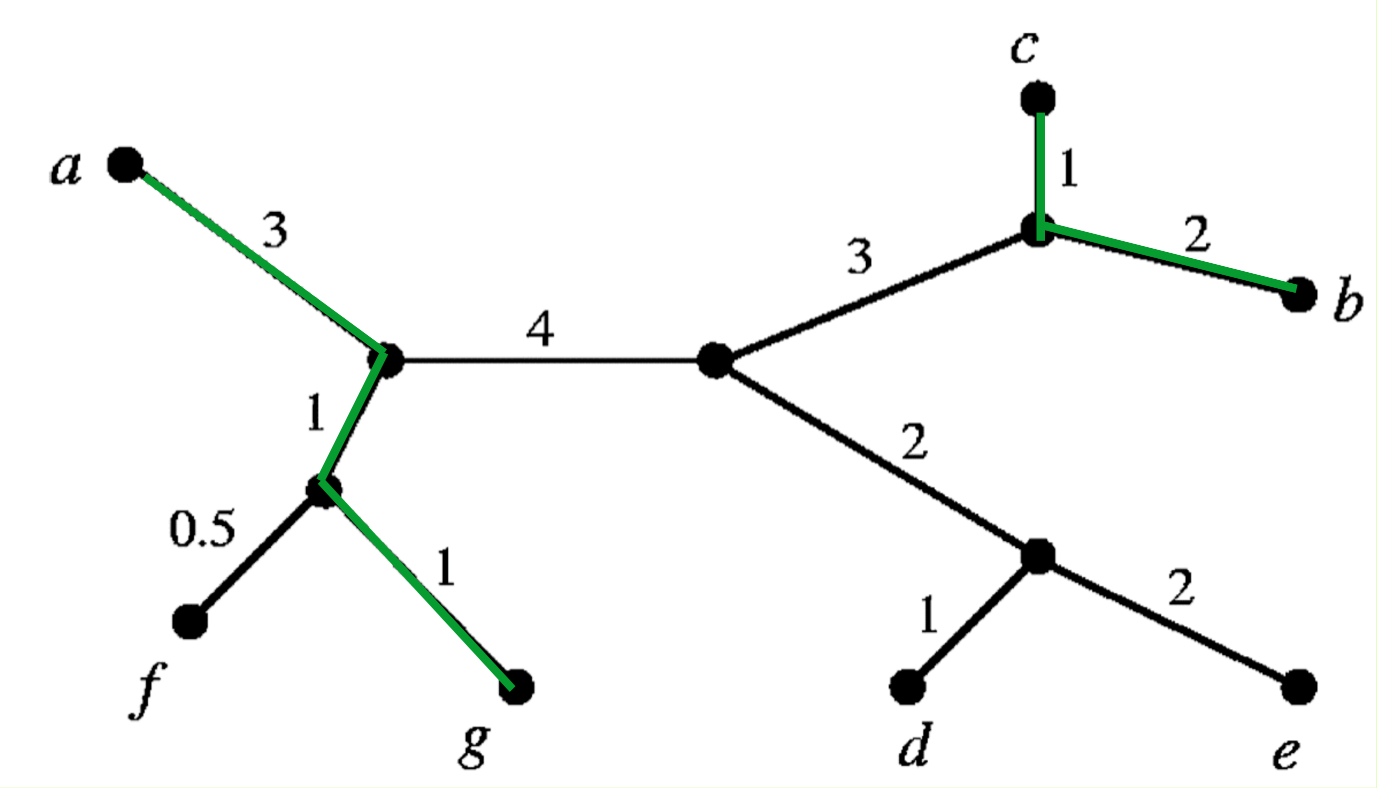




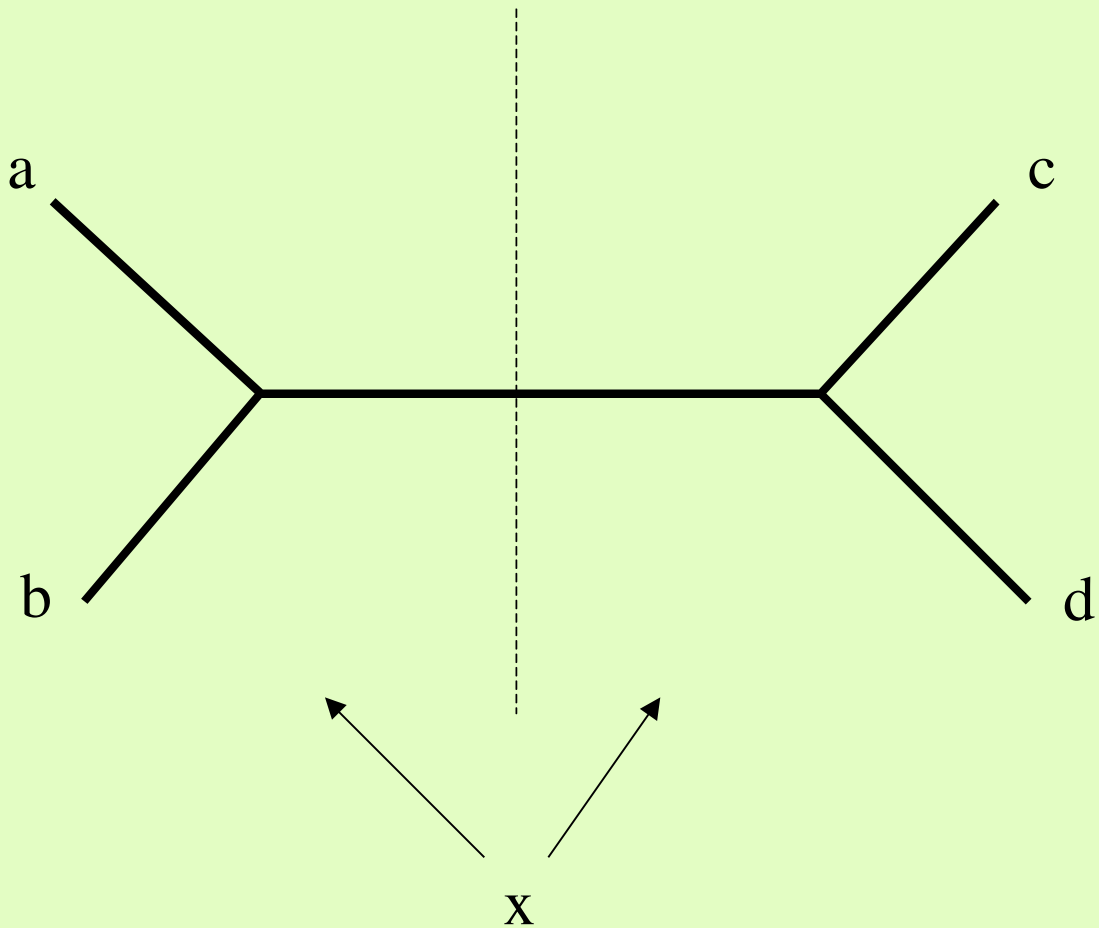
When does a set of weighted quartets correspond exactly to a tree?

- Rules for when a set of *unweighted* quartets correspond to a binary tree, Colonius/Schulze, 1977
- Rules for when set of *weighted* quartets correspond to a binary tree, Dress/Erdös, 2003

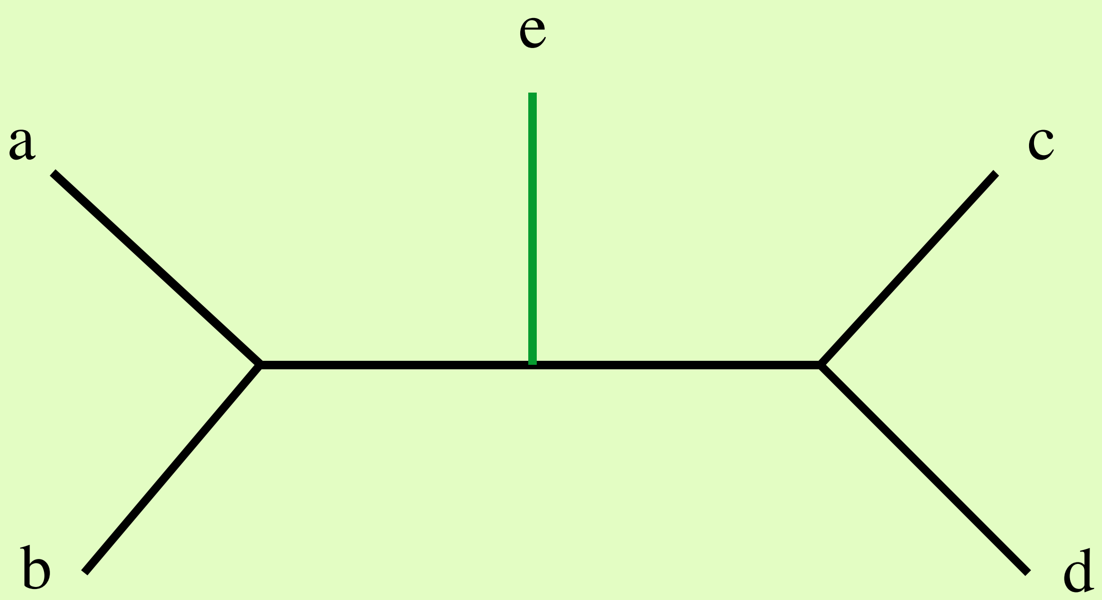
(Q1)[at most 1] For all a,b,c,d in X, at most 1 of w(ab|cd), w(ac|bd), w(ad|bc) is non-zero.



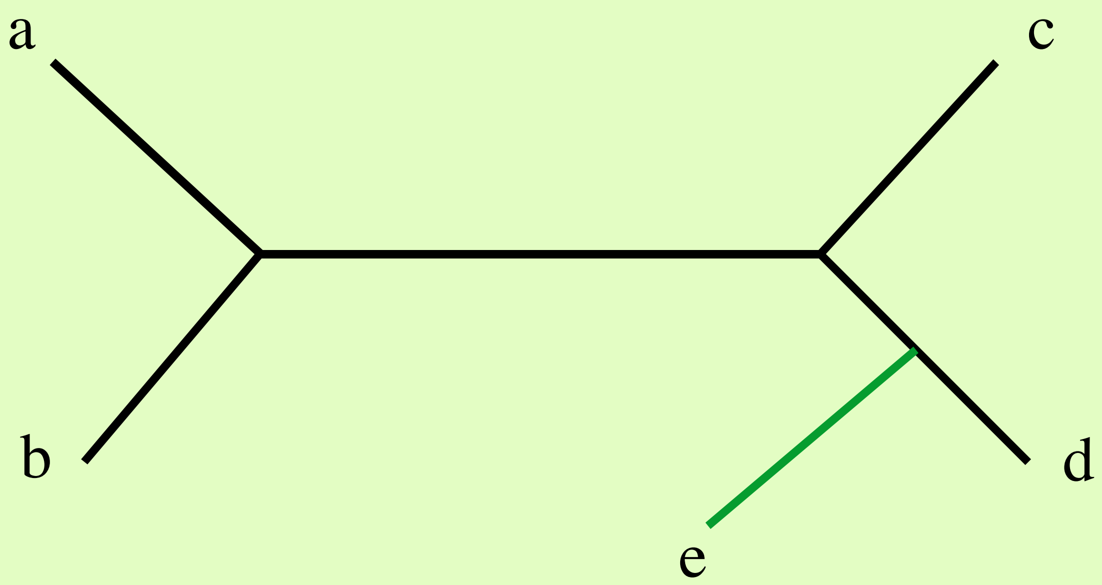
(Q2) For all x in X-{a,b,c,d}, if w(ab|cd) > 0, then either
w(ab|cx) > 0 and w(ab|dx) > 0 or
w(ax|cd) > 0 and w(bx|cd) > 0.



(Q3) For all a,b,c,d,e in X, if w(ab|cd) > w(ab|ce) > 0, then w(ae|cd)=w(ab|cd)-w(ab|ce).



(Q4) For all a,b,c,d,e in X, if $w(ab|cd) > 0$ and $w(bc|de) > 0$, then $w(ab|de) = w(ab|cd) + w(bc|de)$.



Theorem (Grünewald, H., Moulton, Semple, 2007)

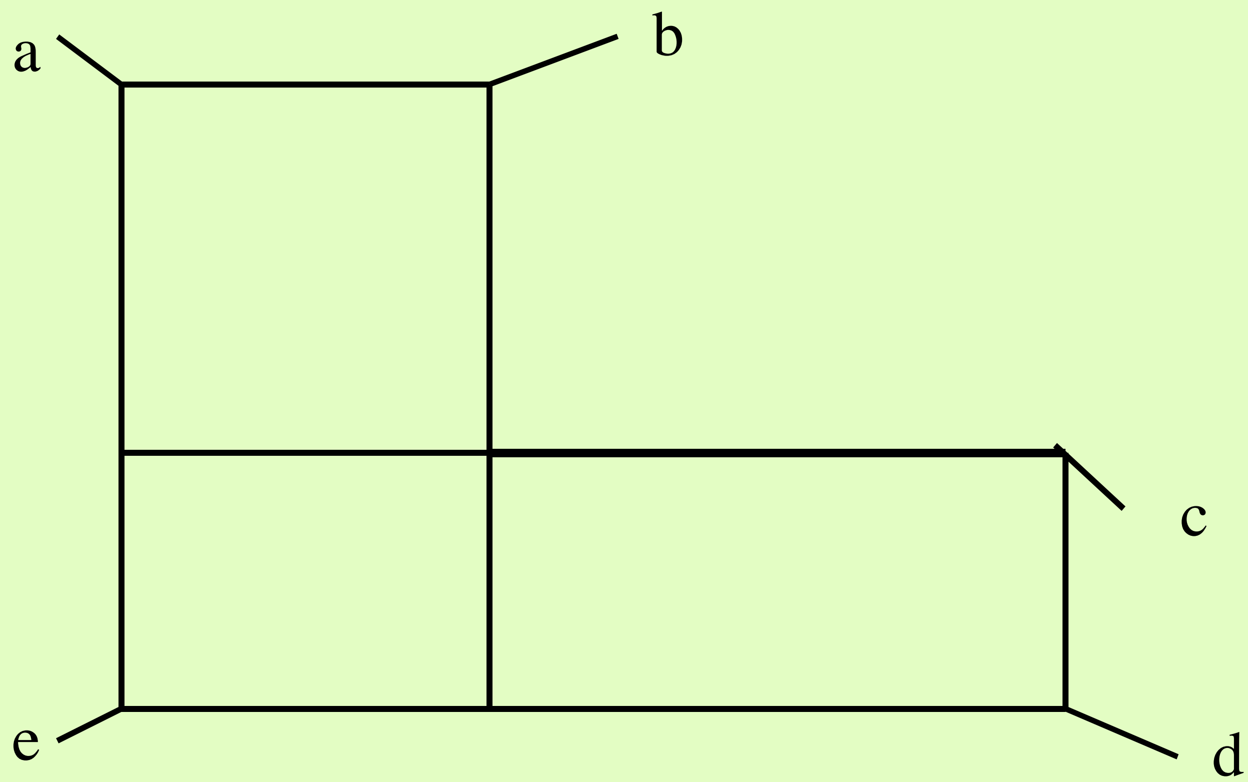
A complete collection Q of weighted quartets is realizable by an edge-weighted phylogenetic tree if and only if Q satisfies $(Q1)^{\text{at most 1}}$-(Q4).

Note

1) If Q is realizable by a tree, then there is only one such tree.

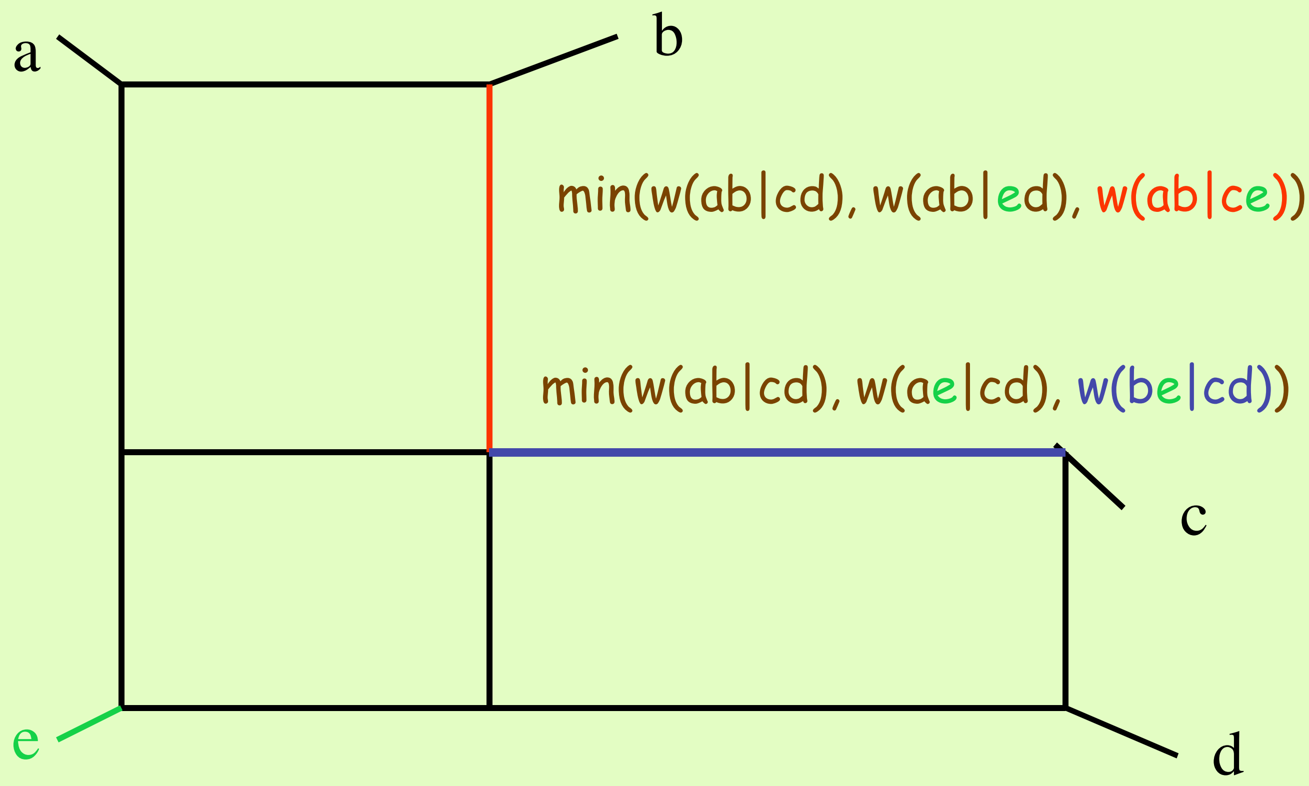
2) If we assume $(Q1)^{\text{precisely 1}}$ i.e. in $(Q1)^{\text{at most 1}}$ we assume precisely one of $w(ab|cd)$, $w(ac|bd)$, $w(ad|bc)$ is zero, then we obtain a *binary* tree.

What should we do if quartets don't fit into a tree, but into ..?



(Q5) For all a,b,c,d,e in X,

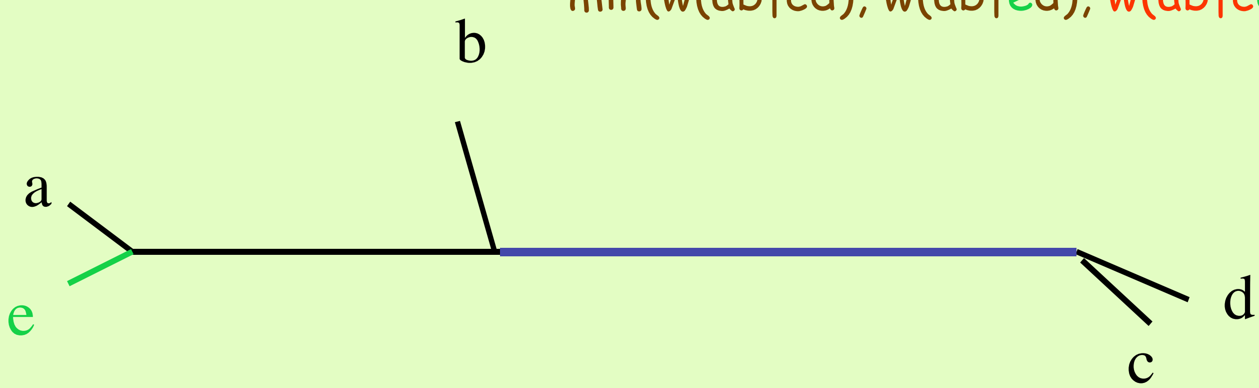
$$w(ab|cd) = \min(w(ab|cd), w(ab|ed), w(ab|ce)) + \min(w(ab|cd), w(ae|cd), w(be|cd)) .$$



(Q5) For all a,b,c,d,e in X,

$$w(ab|cd) = \min(w(ab|cd), w(ab|ed), w(ab|ce)) + \min(w(ab|cd), w(ae|cd), w(be|cd)) .$$

$\min(w(ab|cd), w(ab|ed), w(ab|ce)) = 0$



$\min(w(ab|cd), w(ae|cd), w(be|cd))$
$= w(ab|cd)$

Theorem (Grünewald, H., Moulton, Semple, Spillner)

For a complete collection Q of weighted quartets the following statements hold:

1. Q is realizable by a weighted weakly compatible split system if and only if Q satisfies $(Q1)^{\text{at most 2}}$ and (Q5).
2. Q is realizable by a weighted compatible split system if and only if Q satisfies $(Q1)^{\text{at most 1}}$ and (Q5).
3. Q is realizable by a weighted maximal (= maximum) compatible split system if and only if Q satisfies $(Q1)^{\text{precisely 1}}$ and (Q5).

Regarding:

1. Q is realizable by a weighted weakly compatible split system if and only if Q satisfies (Q1)$^{\text{at most 2}}$ and (Q5):

   if (Q1)$^{\text{precisely 2}}$ then that split system is maximal but need not be maximum.

2. Q is realizable by a weighted compatible split system if and only if Q satisfies (Q1)$^{\text{at most 1}}$ and (Q5):

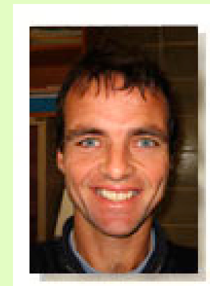
   the corresponding edge-weighted phylogenetic tree need not be binary.

3. Q is realizable by a weighted maximal (= maximum) compatible split system if and only if Q satisfies (Q1)$^{\text{precisely 1}}$ and (Q5):

   the corresponding edge-weighted phylogenetic tree is binary.

Acknowledgements

Stefan Grünewald



Charles Semple

Andreas Spillner

Vincent Moulton