## The complexity of deriving multi-trees from sets of bipartitions



Vincent Moulton, School of Computing Sciences

## Joint work with

- Dr. Katharina Huber, Martin Lott, Dr. Andreas Spillner

School of Computing Sciences,University of East Anglia


- Prof. Bengt Oxelman, Anna Petri

Department of Plant and Environmental Sciences, University of Gothenburg


## Modelling polyploidy

 (multi-trees from networks)

## Networks from multi-trees?



Smedmark et al., Systematic Biology, 2003

## Multiple possibilties



## Aim

Given a multi-labeled tree $T \ldots$...



## Inextendible subtrees



## Construction



Theorem [Huber,Moulton, 2006] $D(T)$ is "minimal" network displaying $T$.

## Question: How do we get the multi-tree?



## Consensus trees


$\mathrm{AB}|\mathrm{CDE}, \mathrm{ABC}| \mathrm{DE} \quad \mathrm{AC}|\mathrm{BDE}, \mathrm{ABC}| \mathrm{DE} \quad \mathrm{AB}|\mathrm{DCE}, \mathrm{ABD}| \mathrm{CE}$


## Problem!

## Theorem

Given a set $S$ of splits of a multi-set $M$, it is NP-hard to decide if $S$ can be displayed by a multi-tree (even if the multiplicity of all elements in $M$ is bounded by 3 ).

## Idea for why this is the case:

$M=\{n x\}$
$S=\left\{n_{l} x\left|\left(n-n_{l}\right) x, \ldots, n_{m} x\right|\left(n-n_{m}\right) x\right\}$


Deciding if we can display this set by a multi-tree is essentially equivalent to deciding if there is a subset of $\left\{n_{1}, . ., n_{m}\right\}$ that adds up to $n / 2$.

## Useful result and conjecture

Given multiset $M=\{m(x) x\}_{x \text { in } X}$, let

$$
\Delta(M)=\Sigma_{x \text { in } X}(m(x)-1) .
$$

Theorem [Lott, Huber, Moulton, Spillner, in press]
If every submultiset of size at most $m:=\max \{2 \Delta, \Delta+2\}$ of a multiset of splits of $M$ can be displayed by a multi-tree, then so can the whole collection.

Conjecture $\quad m=\Delta+2$

## Work in progress...



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