

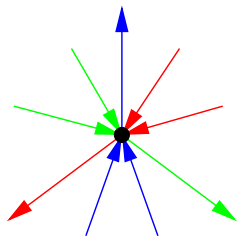
Toroidal Schnyder woods

Benjamin Lévêque

CNRS, LIRMM, Montpellier, France

Schnyder woods (planar triangulations)

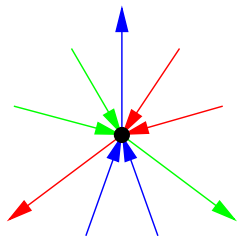
Schnyder (1989)



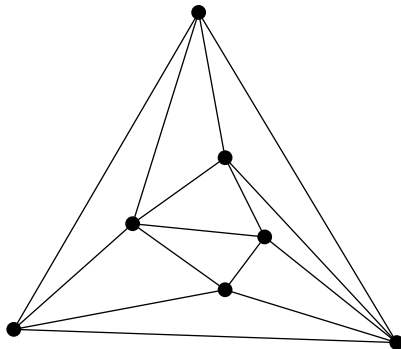
For every inner vertex

Schnyder woods (planar triangulations)

Schnyder (1989)

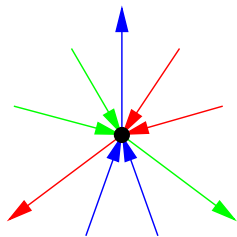


For every inner vertex

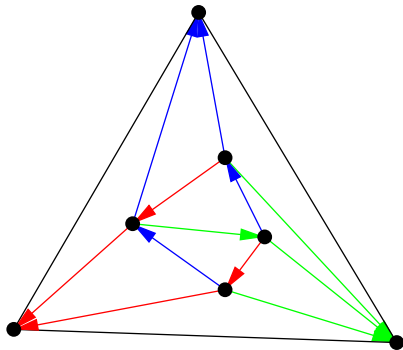


Schnyder woods (planar triangulations)

Schnyder (1989)

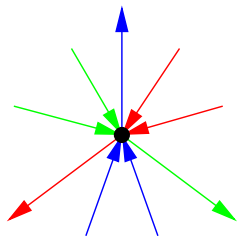


For every inner vertex

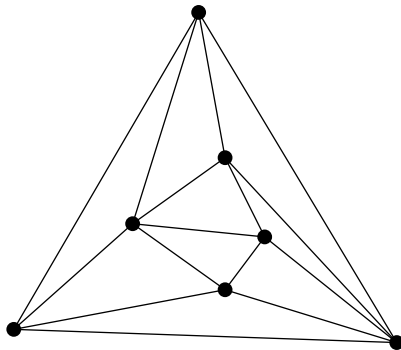


Schnyder woods (planar triangulations)

Schnyder (1989)

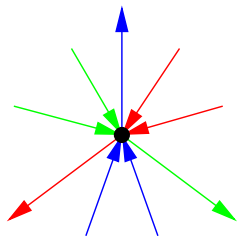


For every inner vertex

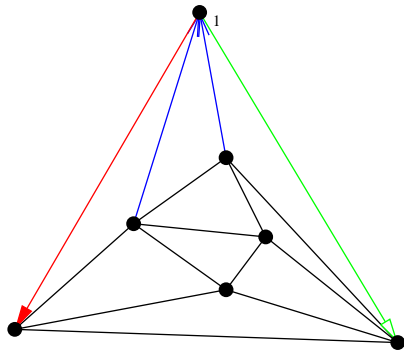


Schnyder woods (planar triangulations)

Schnyder (1989)

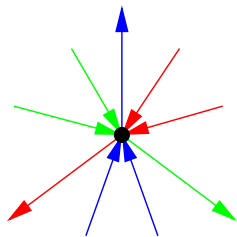


For every inner vertex

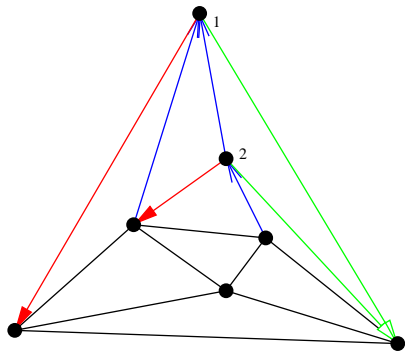


Schnyder woods (planar triangulations)

Schnyder (1989)

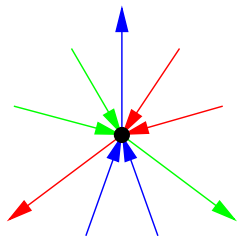


For every inner vertex

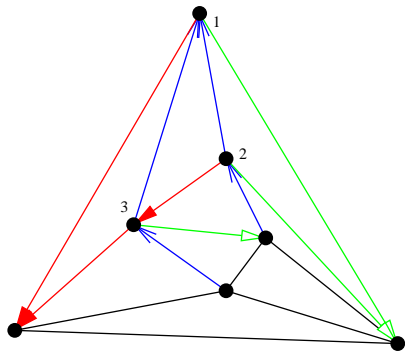


Schnyder woods (planar triangulations)

Schnyder (1989)

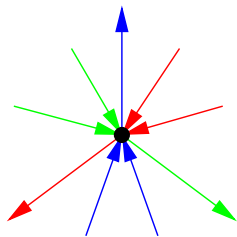


For every inner vertex

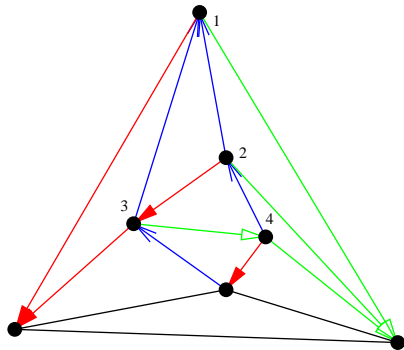


Schnyder woods (planar triangulations)

Schnyder (1989)

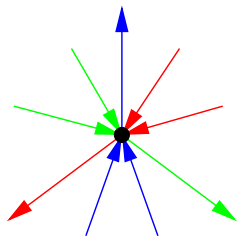


For every inner vertex

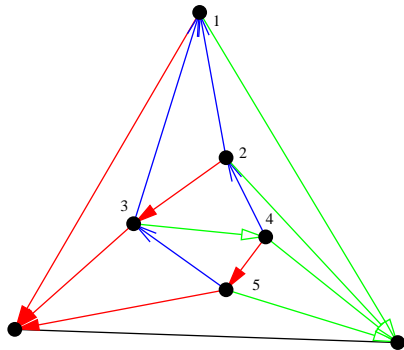


Schnyder woods (planar triangulations)

Schnyder (1989)

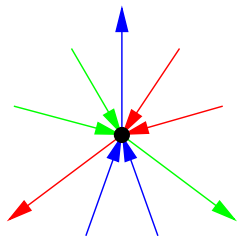


For every inner vertex

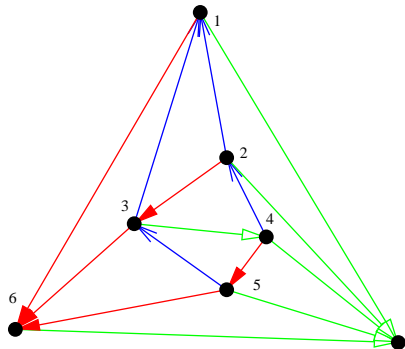


Schnyder woods (planar triangulations)

Schnyder (1989)

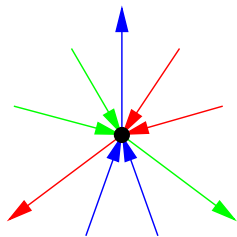


For every inner vertex

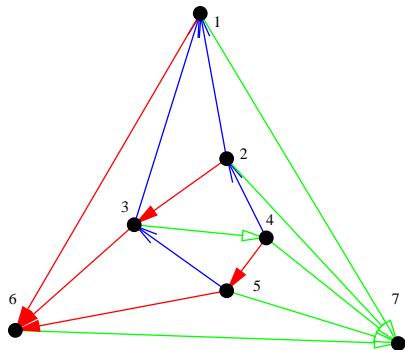


Schnyder woods (planar triangulations)

Schnyder (1989)

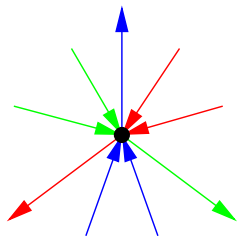


For every inner vertex

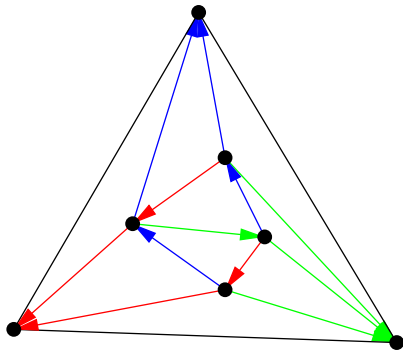


Schnyder woods (planar triangulations)

Schnyder (1989)

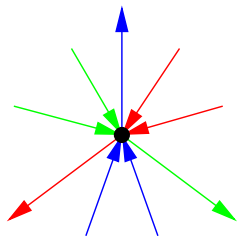


For every inner vertex

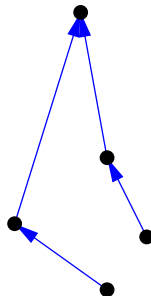


Schnyder woods (planar triangulations)

Schnyder (1989)

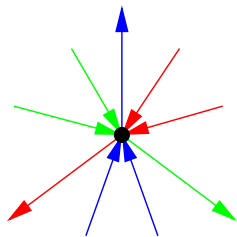


For every inner vertex

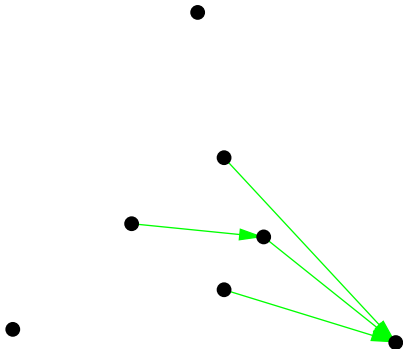


Schnyder woods (planar triangulations)

Schnyder (1989)

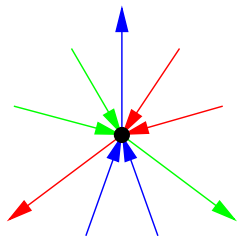


For every inner vertex

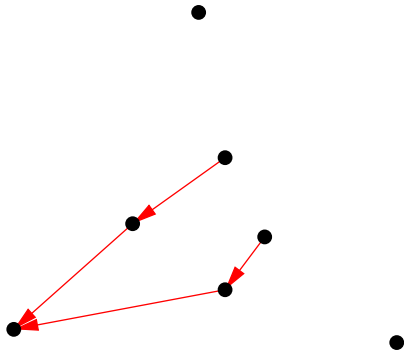


Schnyder woods (planar triangulations)

Schnyder (1989)

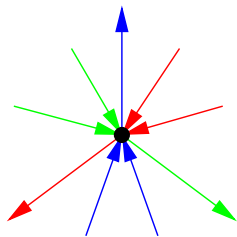


For every inner vertex

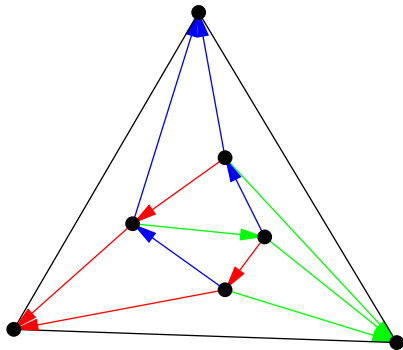


Schnyder woods (planar triangulations)

Schnyder (1989)

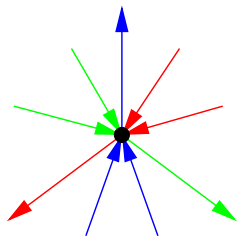


For every inner vertex

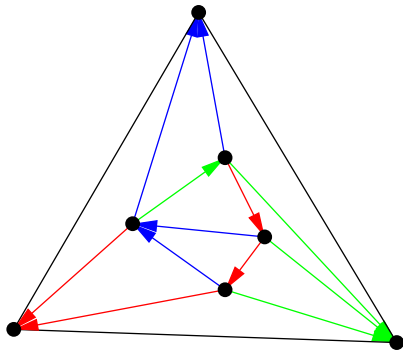


Schnyder woods (planar triangulations)

Schnyder (1989)

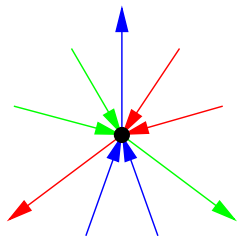


For every inner vertex

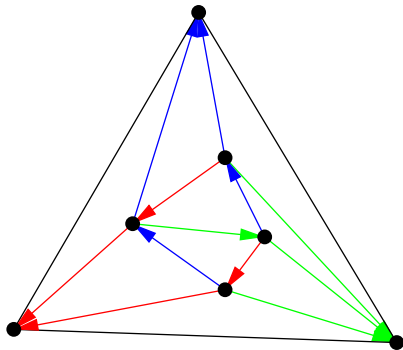


Schnyder woods (planar triangulations)

Schnyder (1989)

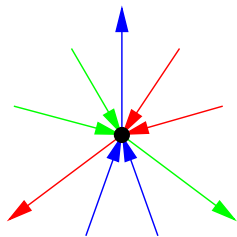


For every inner vertex

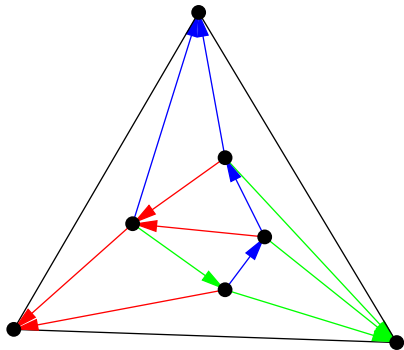


Schnyder woods (planar triangulations)

Schnyder (1989)

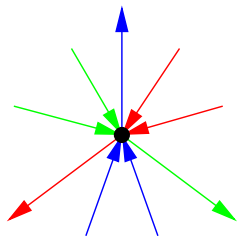


For every inner vertex

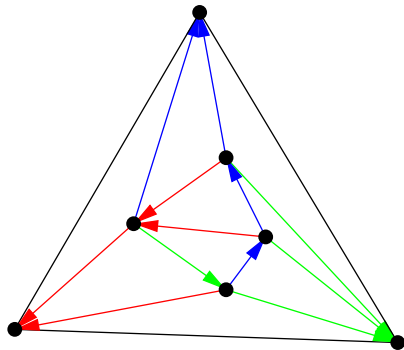


Schnyder woods (planar triangulations)

Schnyder (1989)



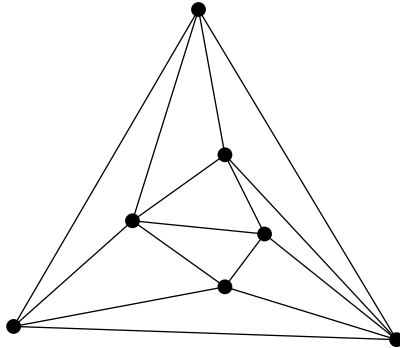
For every inner vertex



Planar graph \iff Incident poset has DM-dimension ≤ 3

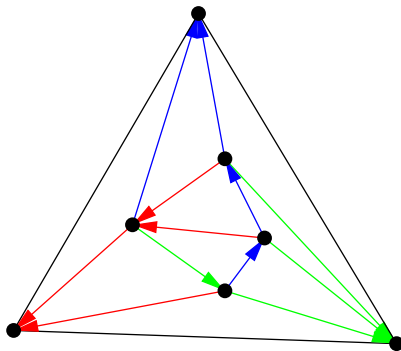
Optimal encoding

Poulalhon and Schaeffer (2003)



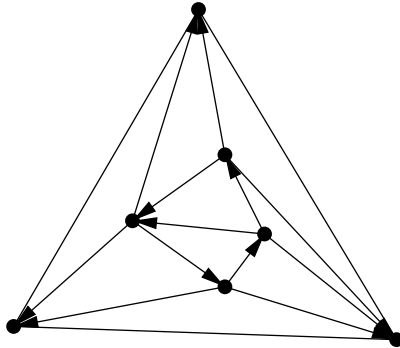
Optimal encoding

Poulalhon and Schaeffer (2003)



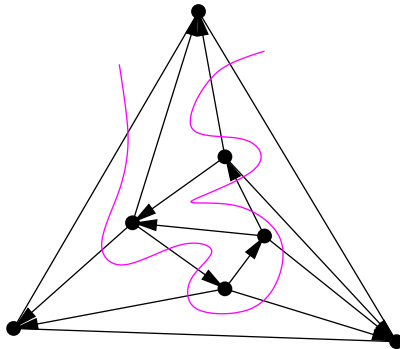
Optimal encoding

Poulalhon and Schaeffer (2003)



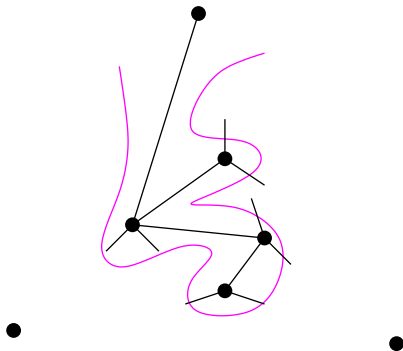
Optimal encoding

Poulalhon and Schaeffer (2003)



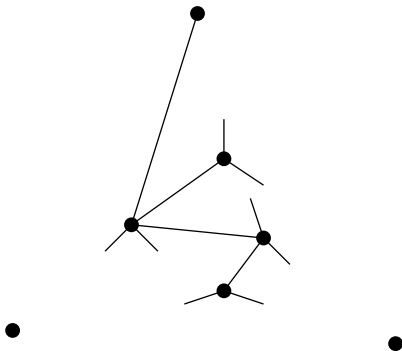
Optimal encoding

Poulalhon and Schaeffer (2003)



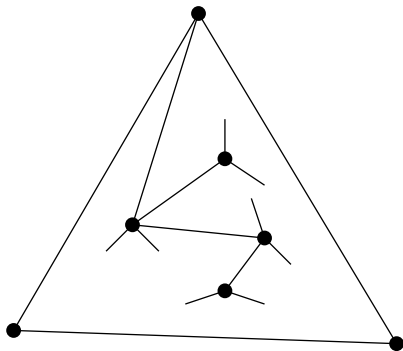
Optimal encoding

Poulalhon and Schaeffer (2003)



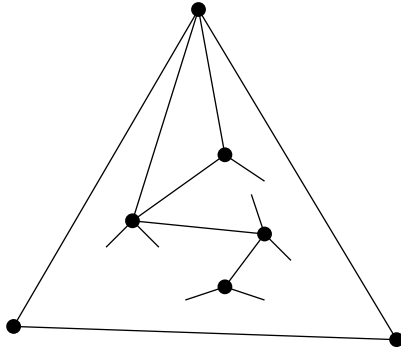
Optimal encoding

Poulalhon and Schaeffer (2003)



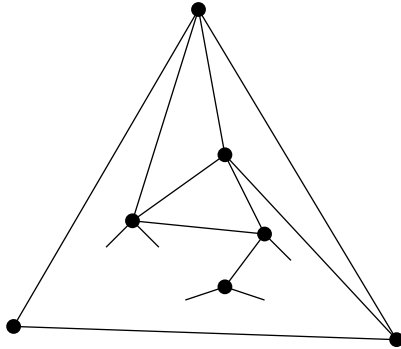
Optimal encoding

Poulalhon and Schaeffer (2003)



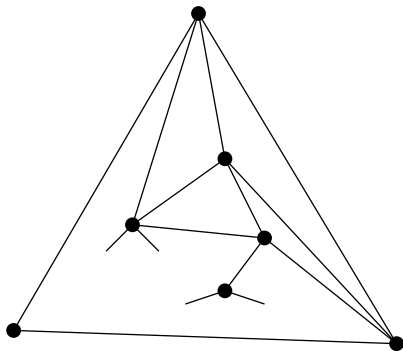
Optimal encoding

Poulalhon and Schaeffer (2003)



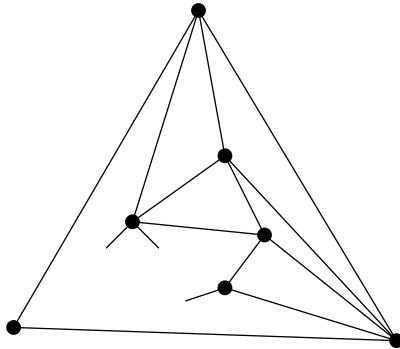
Optimal encoding

Poulalhon and Schaeffer (2003)



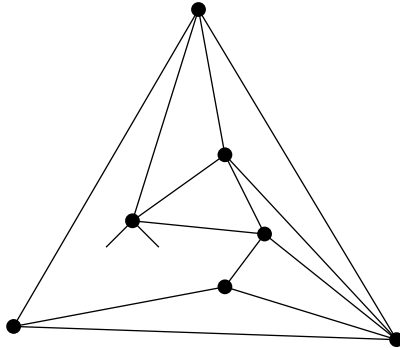
Optimal encoding

Poulalhon and Schaeffer (2003)



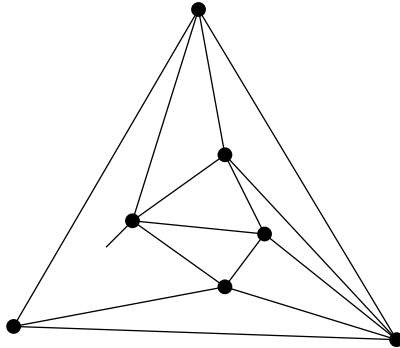
Optimal encoding

Poulalhon and Schaeffer (2003)



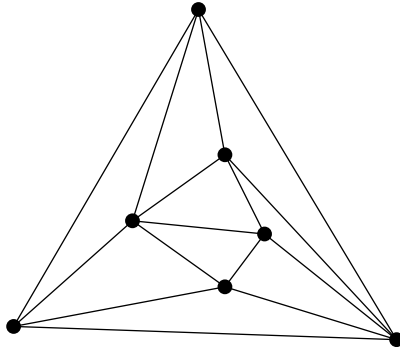
Optimal encoding

Poulalhon and Schaeffer (2003)



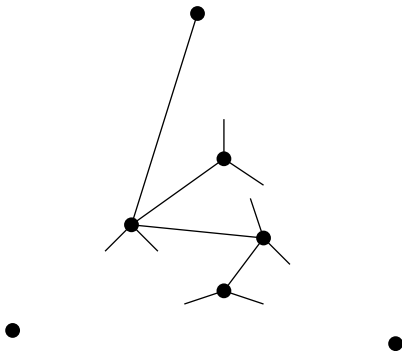
Optimal encoding

Poulalhon and Schaeffer (2003)



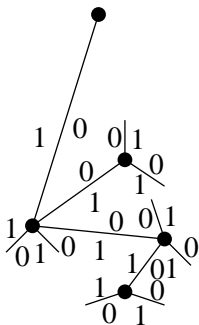
Optimal encoding

Poulalhon and Schaeffer (2003)



Optimal encoding

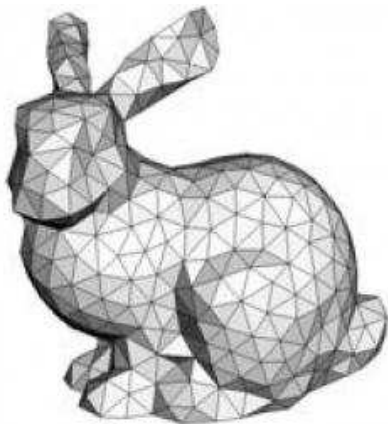
Poulalhon and Schaeffer (2003)



110101110100101001101000 \rightsquigarrow 6n bits

Optimal encoding

Poulalhon and Schaeffer (2003)



1001100000010000 \rightsquigarrow 4n bits (n bits 1) \rightsquigarrow 3,25n bits

...1w10w10w0...

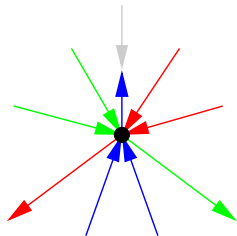
OPTIMAL !

Generalization to 3-connected planar map

Felsner (2001), Miller (2002)

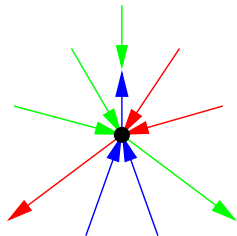
Generalization to 3-connected planar map

Felsner (2001), Miller (2002)



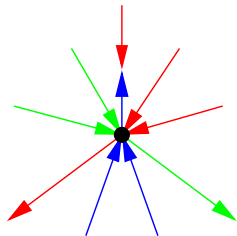
Generalization to 3-connected planar map

Felsner (2001), Miller (2002)



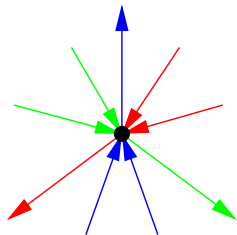
Generalization to 3-connected planar map

Felsner (2001), Miller (2002)



Generalization to 3-connected planar map

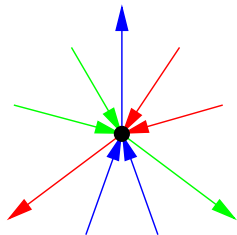
Felsner (2001), Miller (2002)



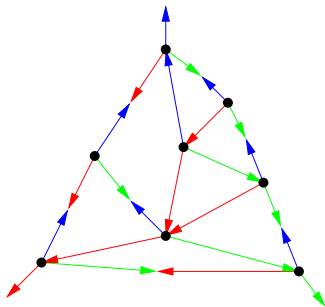
For every vertex

Generalization to 3-connected planar map

Felsner (2001), Miller (2002)

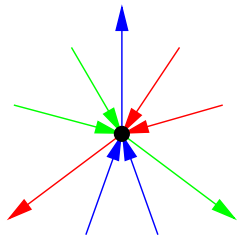


For every vertex

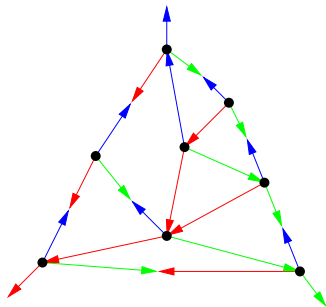


Generalization to 3-connected planar map

Felsner (2001), Miller (2002)



For every vertex

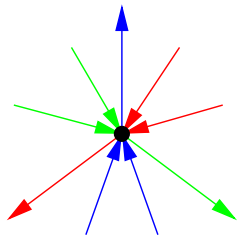


AND

No monochromatic cycles

Generalization to (internally) 3-connected planar map

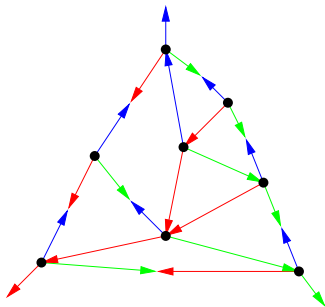
Felsner (2001), Miller (2002)



For every vertex

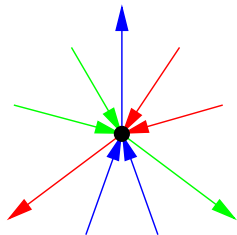
AND

No monochromatic cycles

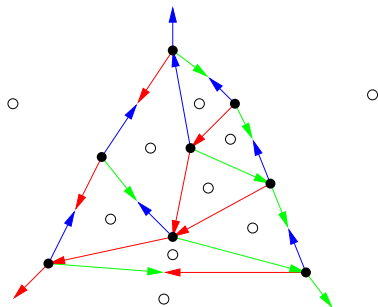


Generalization to (internally) 3-connected planar map

Felsner (2001), Miller (2002)



For every vertex

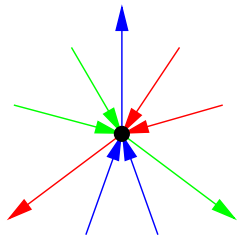


AND

No monochromatic cycles

Generalization to (internally) 3-connected planar map

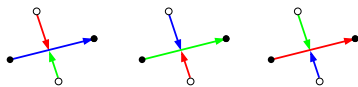
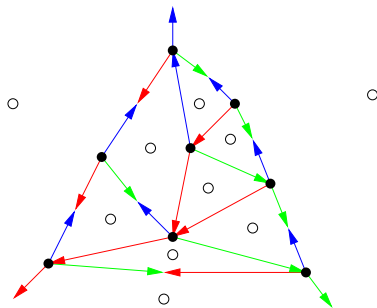
Felsner (2001), Miller (2002)



For every vertex

AND

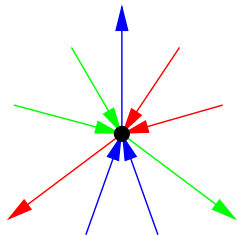
No monochromatic cycles



Dual rules

Generalization to (internally) 3-connected planar map

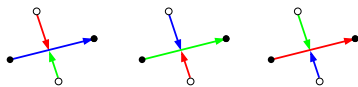
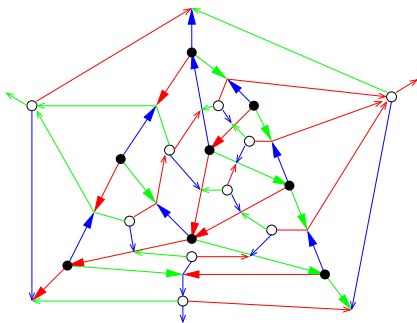
Felsner (2001), Miller (2002)



For every vertex

AND

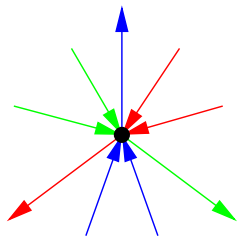
No monochromatic cycles



Dual rules

Generalization to (internally) 3-connected planar map

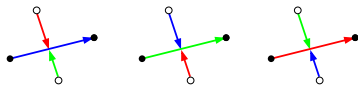
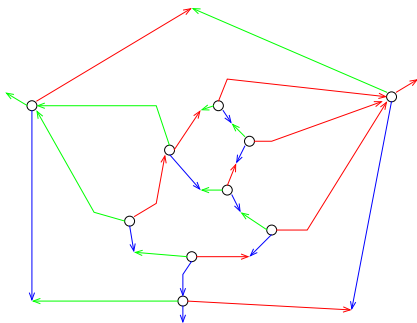
Felsner (2001), Miller (2002)



For every vertex

AND

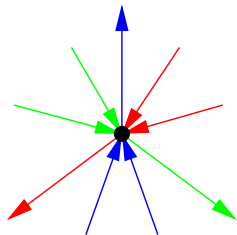
No monochromatic cycles



Dual rules

Generalization to (internally) 3-connected planar map

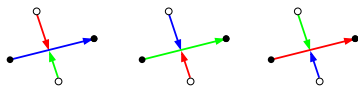
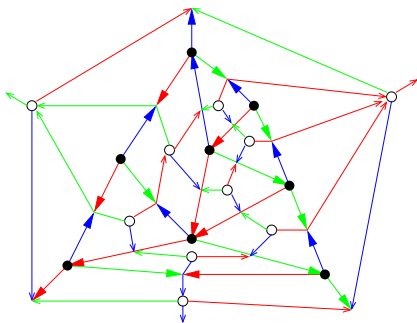
Felsner (2001), Miller (2002)



For every vertex

AND

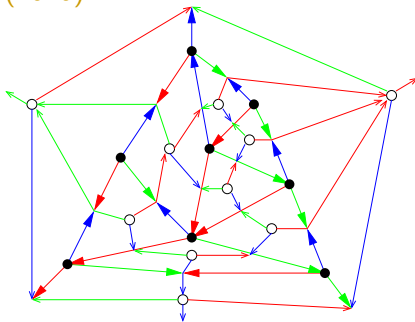
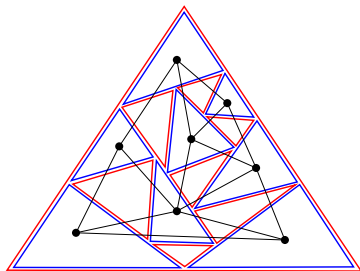
No monochromatic cycles



Dual rules

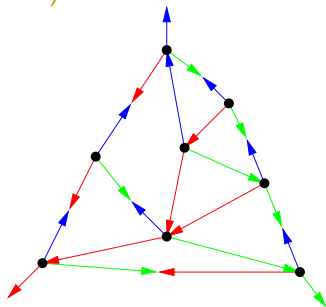
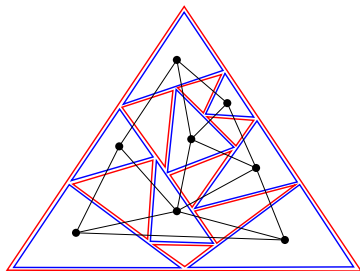
Generalization to (internally) 3-connected planar map

Gonçalves, Lévêque, Pinlou (2010)



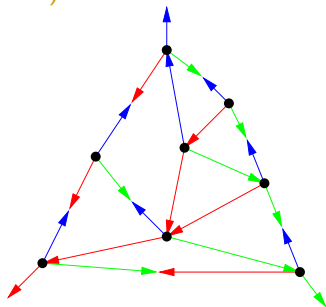
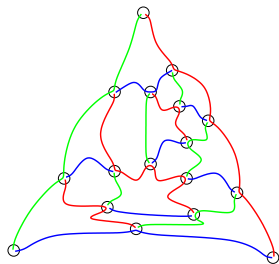
Generalization to (internally) 3-connected planar map

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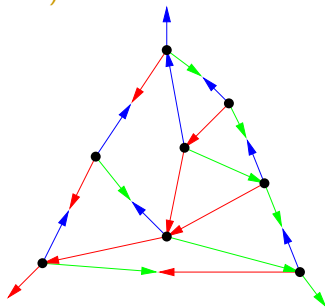
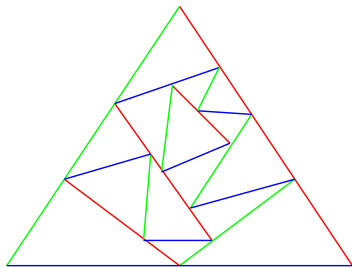
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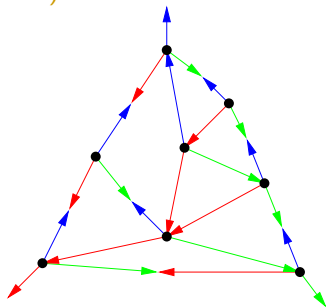
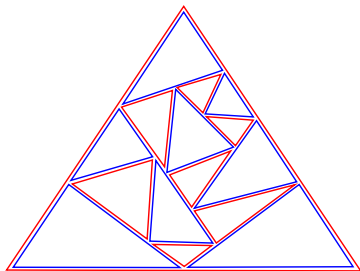
Generalization to (internally) 3-connected planar map

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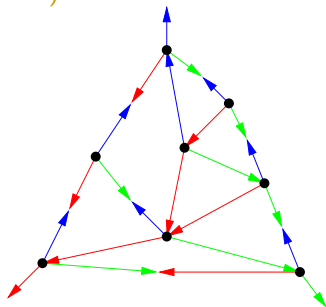
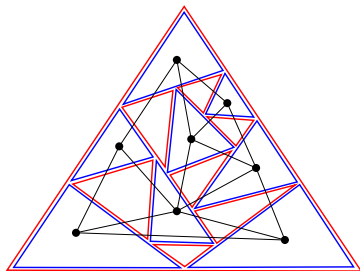
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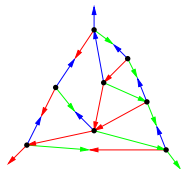
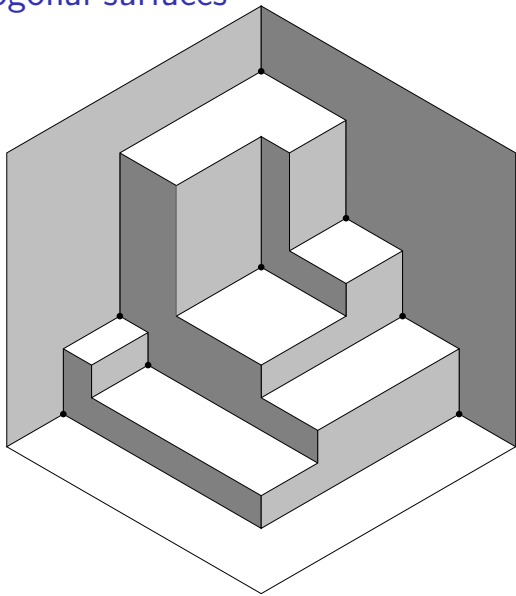


Generalization to (internally) 3-connected planar map

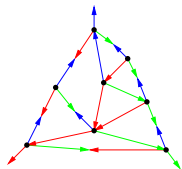
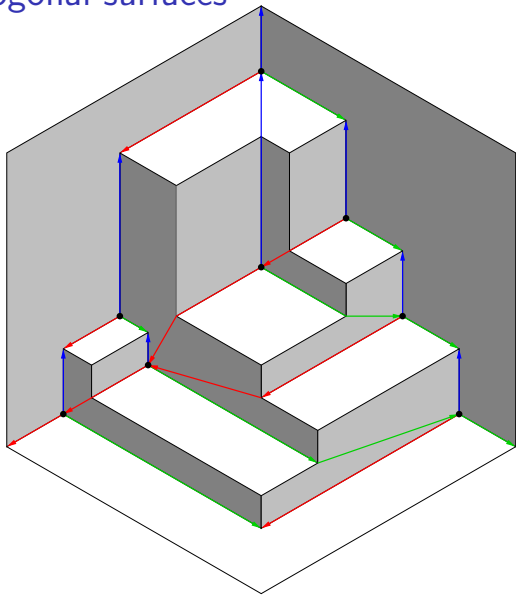
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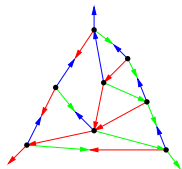
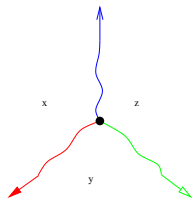
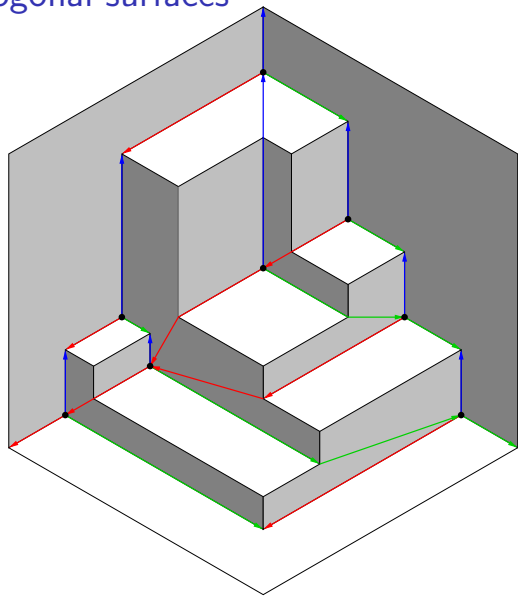
Orthogonal surfaces



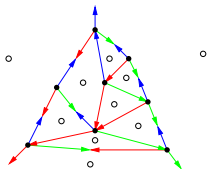
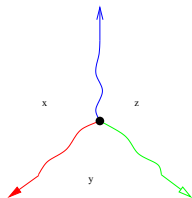
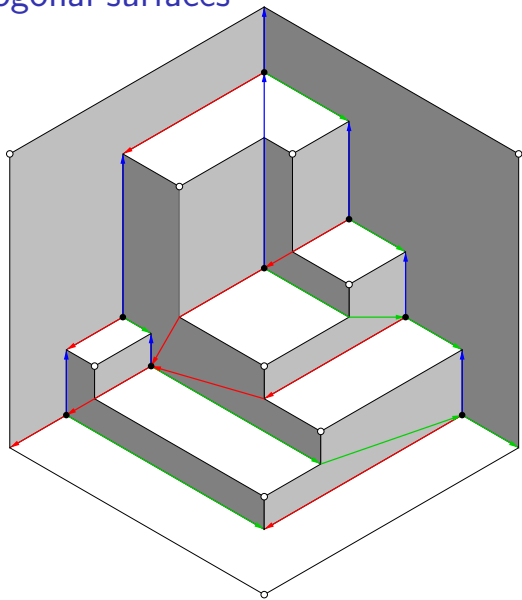
Orthogonal surfaces



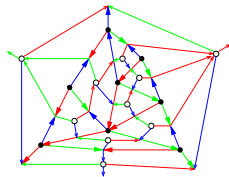
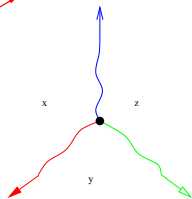
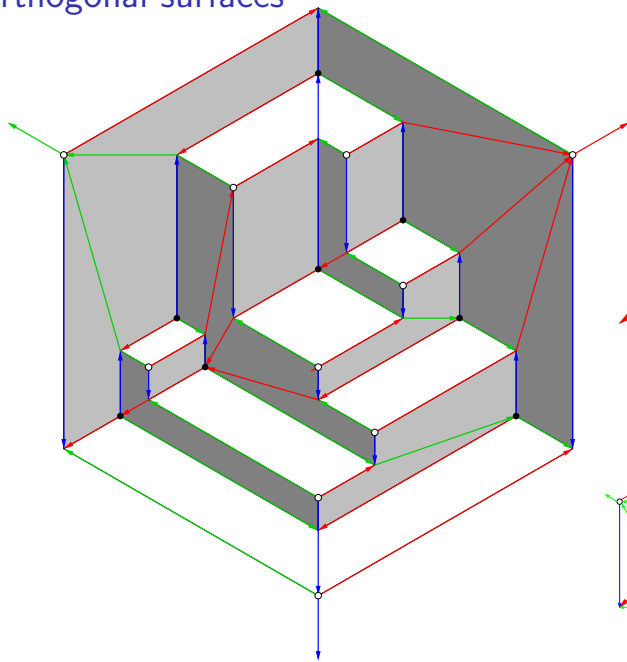
Orthogonal surfaces



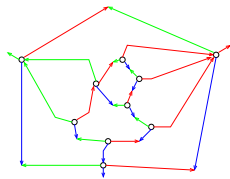
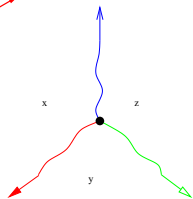
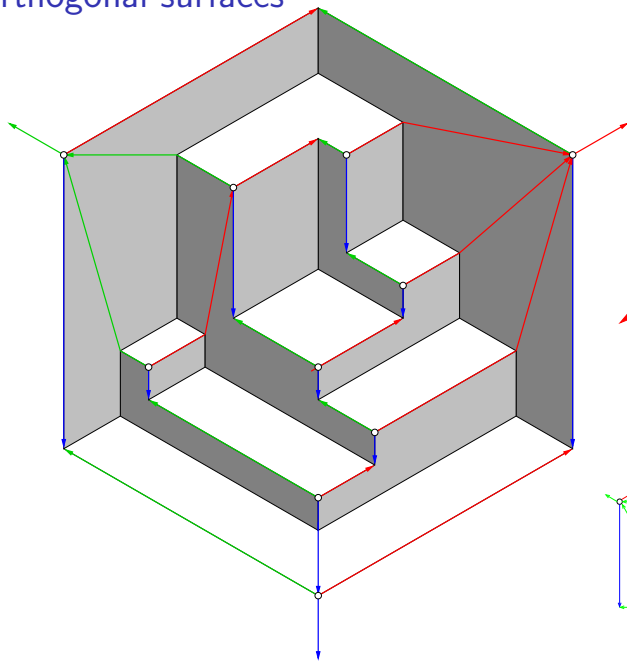
Orthogonal surfaces



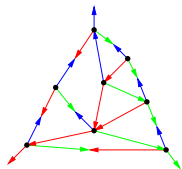
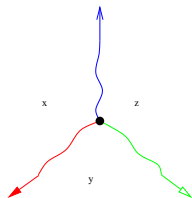
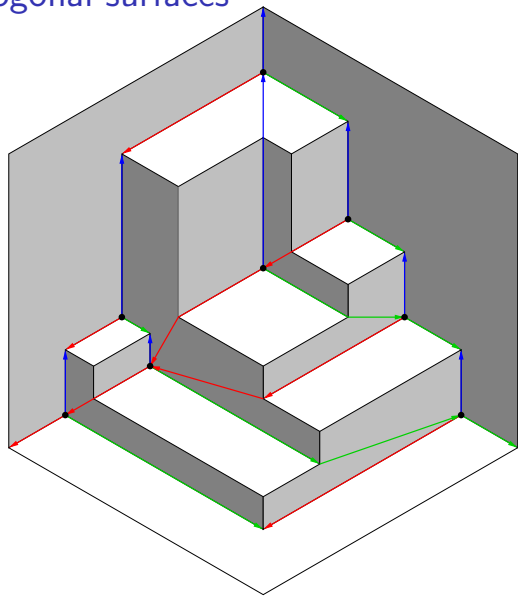
Orthogonal surfaces



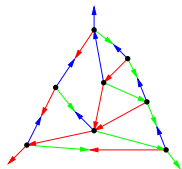
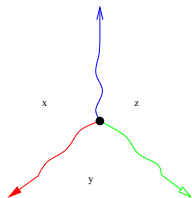
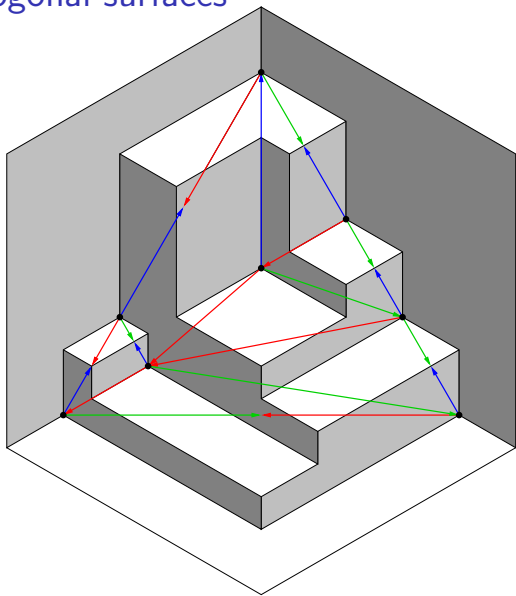
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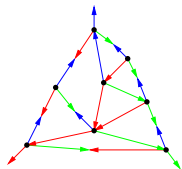
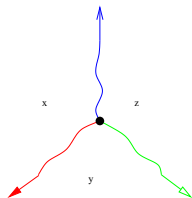
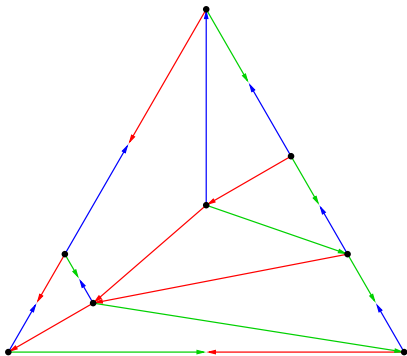
Orthogonal surfaces



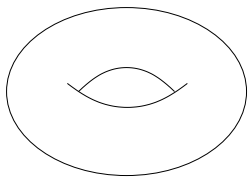
Orthogonal surfaces



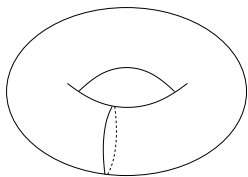
Orthogonal surfaces



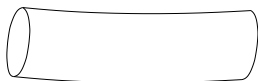
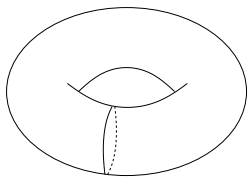
The torus



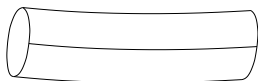
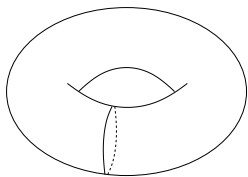
The torus



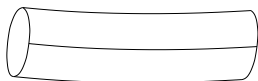
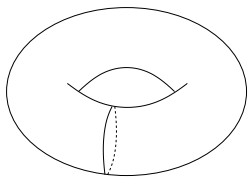
The torus



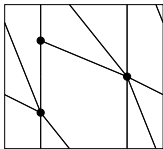
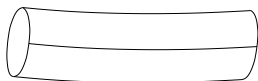
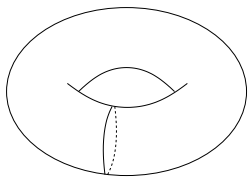
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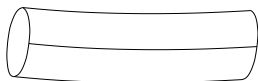
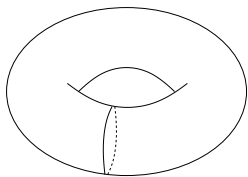
The torus



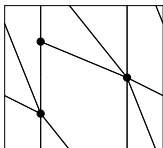
The torus



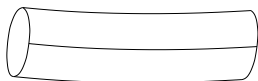
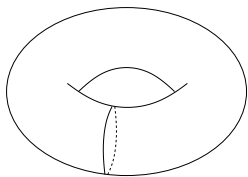
The torus



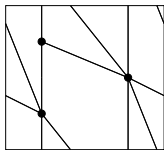
No loop and no multiple edges



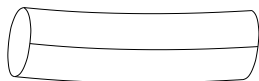
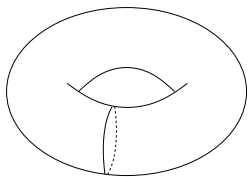
The torus



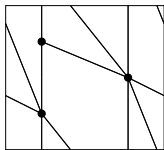
No **contractible** loop and no multiple edges



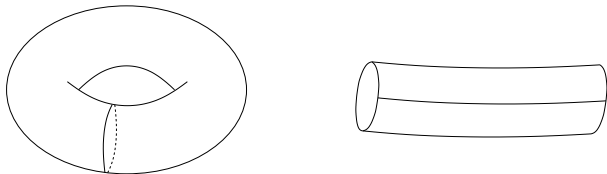
The torus



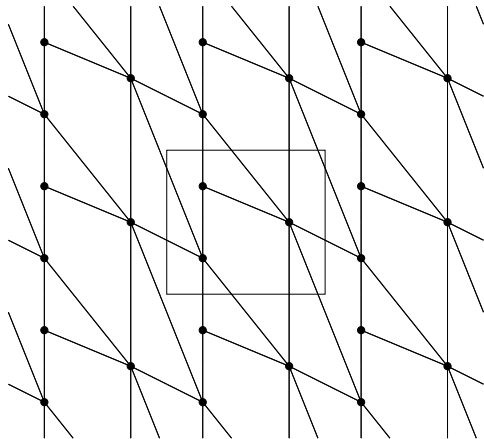
No **contractible** loop and no **homotopic** multiple edges



The torus



No **contractible** loop and no **homotopic** multiple edges



A generalization of Schnyder woods to higher genus

A generalization of Schnyder woods to higher genus

Castelli Aleardi, Fusy, Lewiner (2009) : applications to encoding

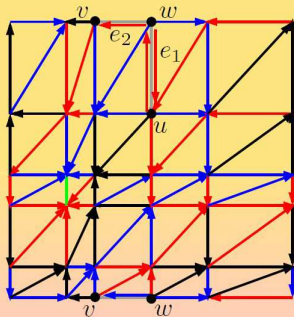
i) a small set \mathcal{E}^s of *special* edges,
doubly oriented and colored

(u, v, w)

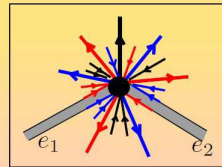
at most $2 \cdot 2g$ multiple vertices
(incident to special edges)



$\mathcal{E}^s = \{e_1, e_2\}$



ii) a new local condition for
edges in a sector incident to
a multiple vertex



A different approach for the torus

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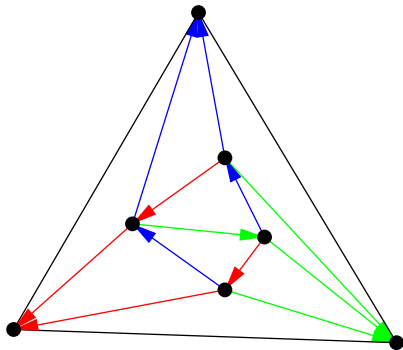
	Plane	Torus
Euler's Formula	$n - m + f = 2$	$n - m + f = 0$

A different approach for the torus

	Plane	Torus
Euler's Formula	$n - m + f = 2$	$n - m + f = 0$
Triangulation	$m = 3n - 6$	

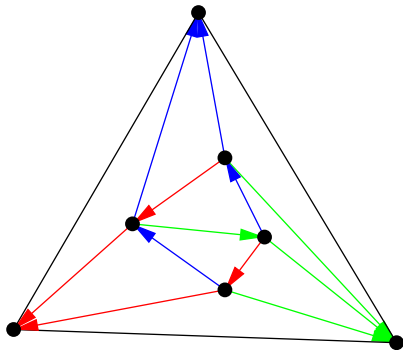
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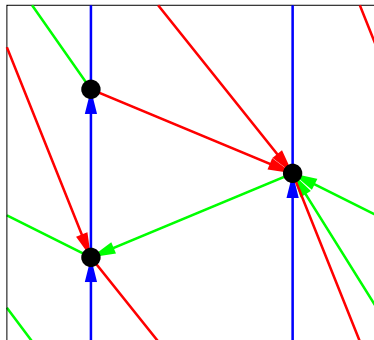
A different approach for the torus

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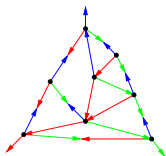


Definition of (generalized) Schnyder wood on the Torus

Definition of (generalized) Schnyder wood on the Torus

Planar Schnyder wood Felsner (2001)

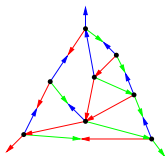
- ▶ Schnyder property + half edge.
- ▶ No monochromatic cycle.



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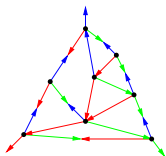


Toroidal Schnyder wood Gonçalves, Lévêque (2011)

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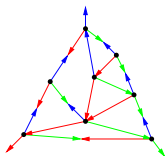
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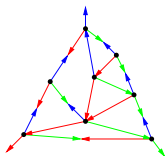
Toroidal Schnyder wood Gonçalves, Lévêque (2011)

- ▶ Schnyder property.
- ▶ No contractible monochromatic cycles ?

Definition of (generalized) Schnyder wood on the Torus

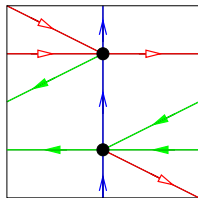
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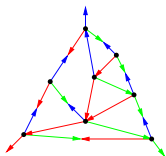
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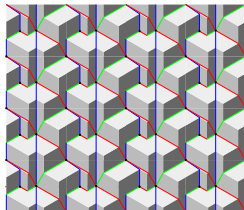
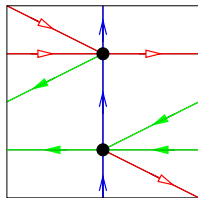
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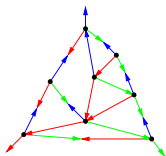
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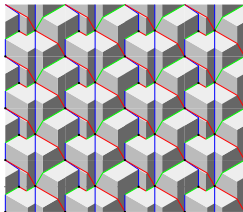
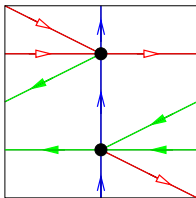
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Toroidal Schnyder wood Gonçalves, Lévêque (2011)

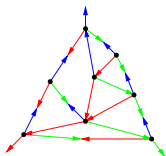
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Definition of (generalized) Schnyder wood on the Torus

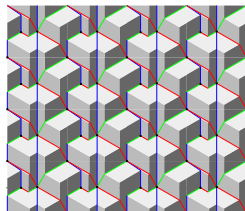
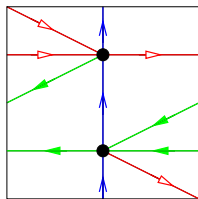
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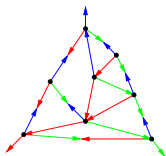
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- ▶ Every monochromatic cycle intersects at least one monochromatic cycle of each color.



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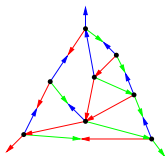
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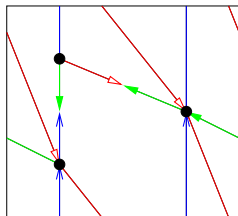
Planar Schnyder wood Felsner (2001)

- ▶ Schnyder property + half edge.
- ▶ No monochromatic cycle.



Toroidal Schnyder wood Gonçalves, Lévêque (2011)

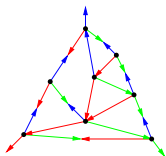
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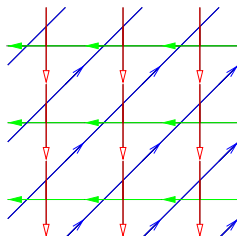
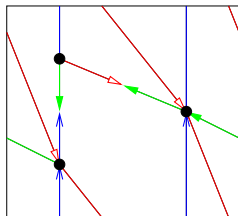
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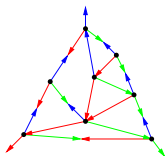
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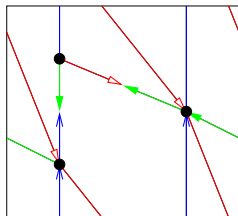
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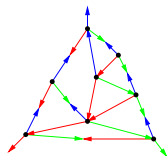
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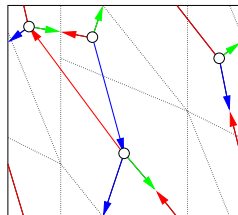
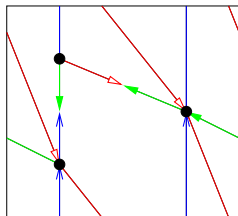
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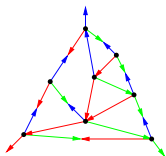
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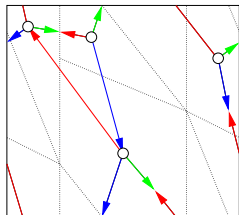
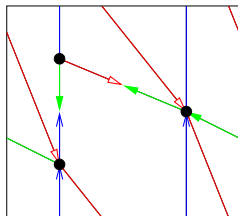
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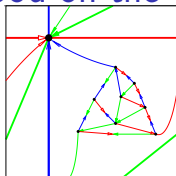


Theorem A toroidal map admits a Schnyder wood if and only if it is essentially 3-connected.

Definition of (generalized) Schnyder wood on the Torus

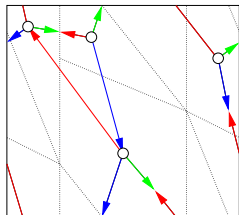
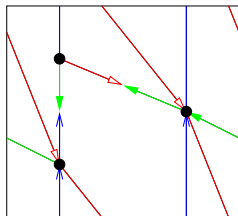
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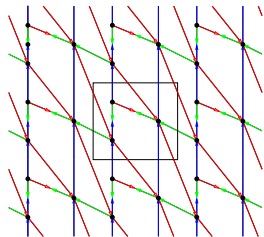
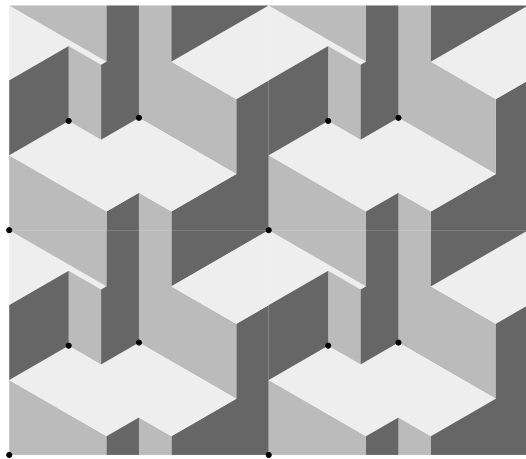
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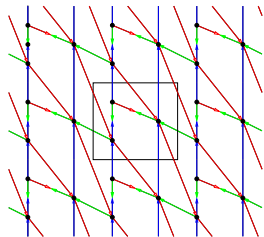
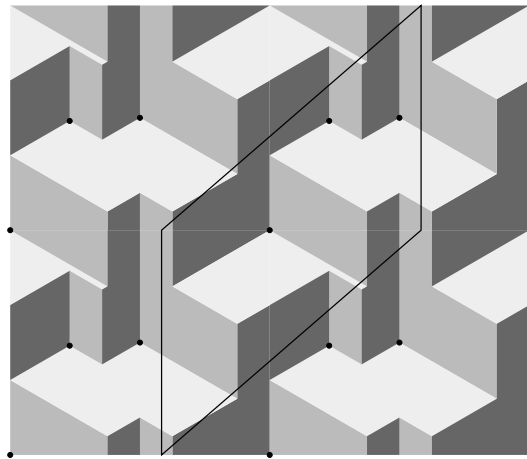


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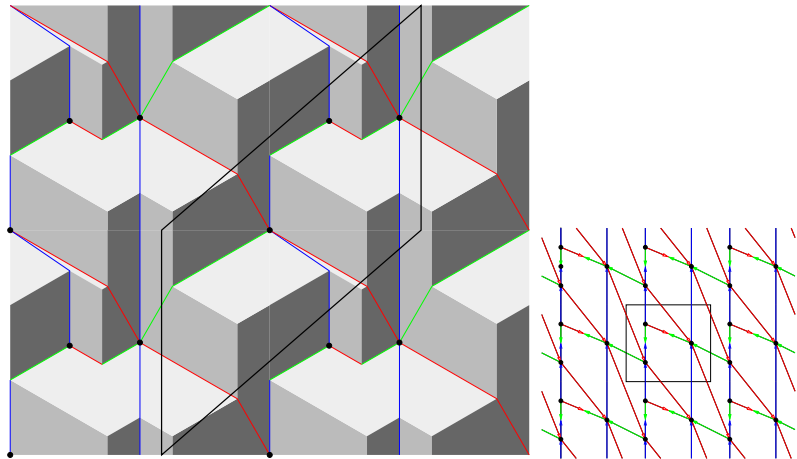
Orthogonal surfaces



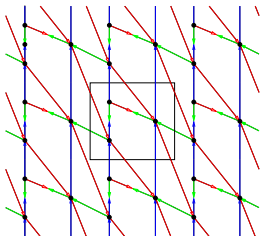
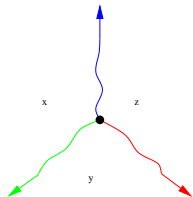
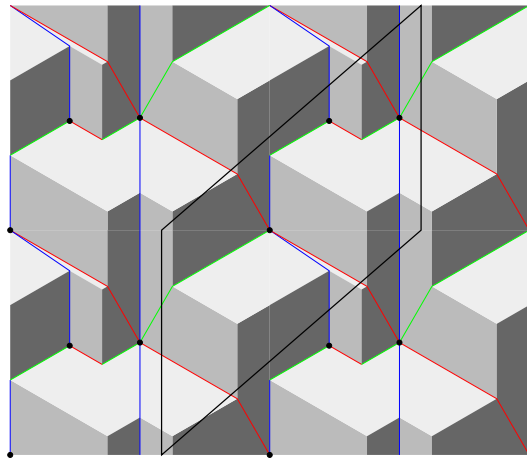
Orthogonal surfaces



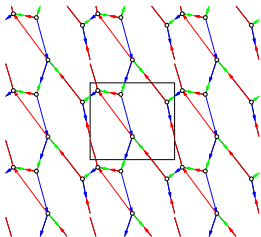
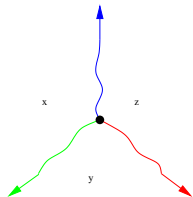
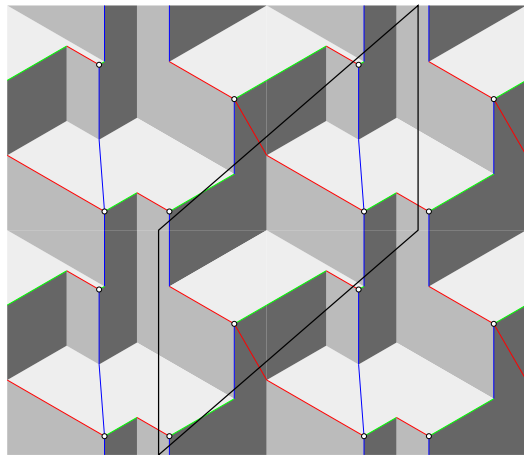
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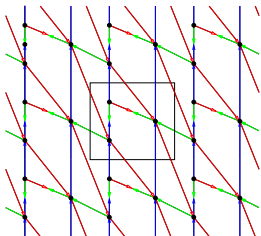
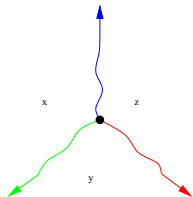
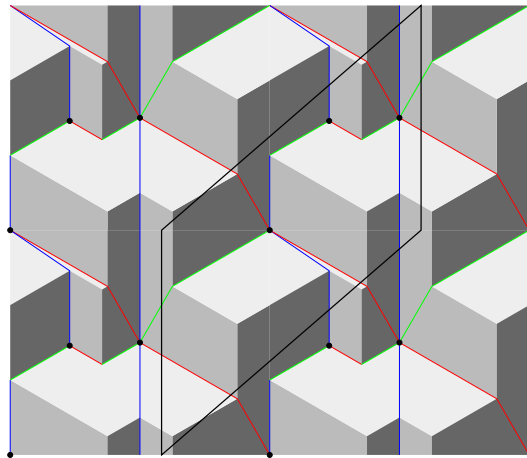
Orthogonal surfaces



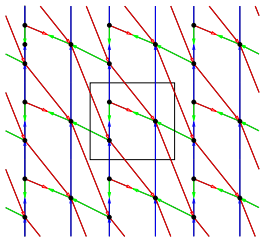
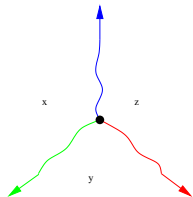
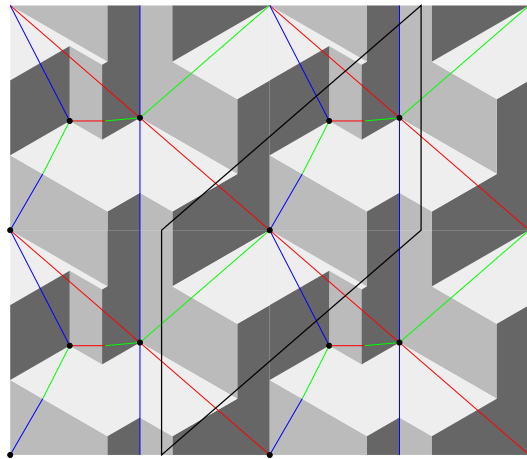
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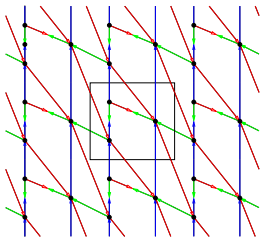
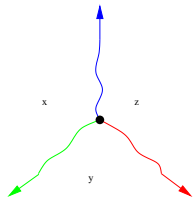
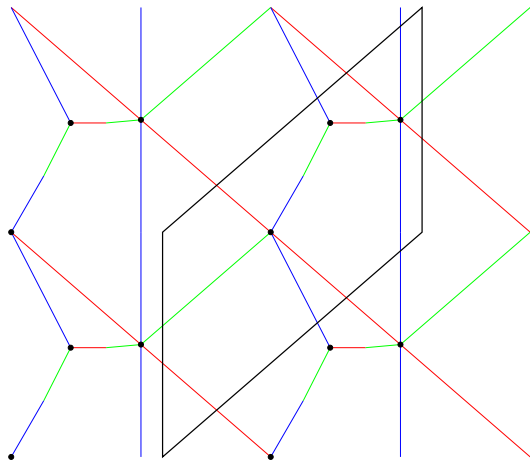
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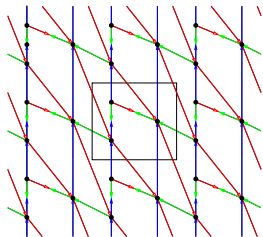
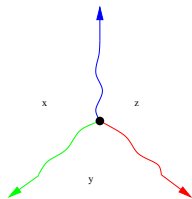
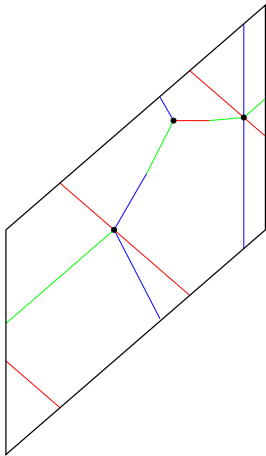
Orthogonal surfaces



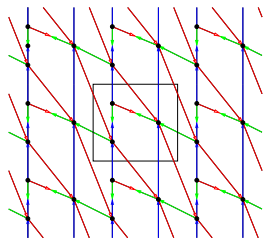
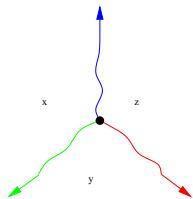
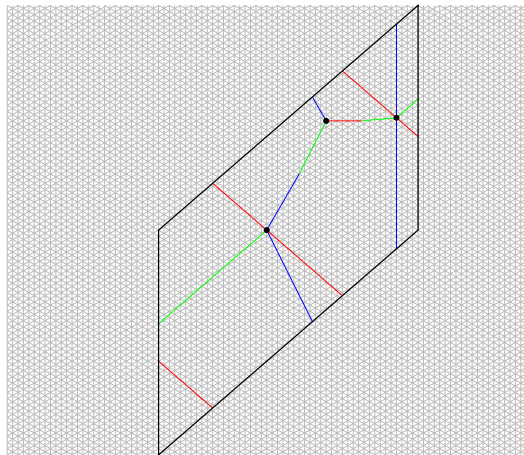
Orthogonal surfaces



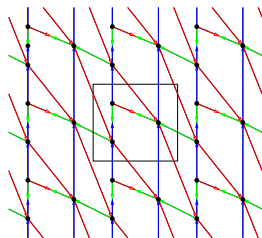
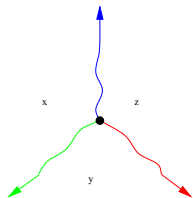
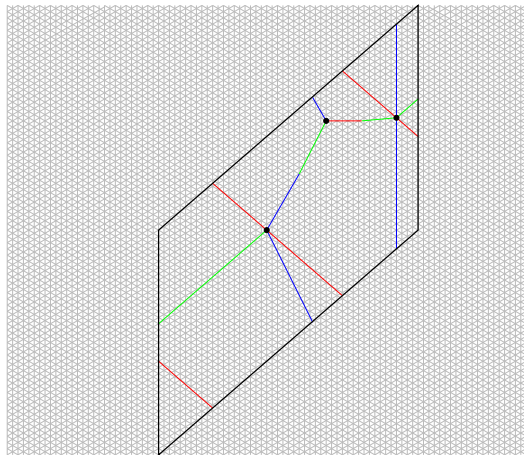
Orthogonal surfaces



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Orthogonal surfaces



Theorem A simple toroidal map admits a straight line representation in a grid of size $\mathcal{O}(n^2) \times \mathcal{O}(n^2)$

Open questions

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- ▶ Transversal structures, 2-orientations, . . .
- ▶ Higher genus, Higher dimension