The k-Sparsest Subgraph Problem in (Proper) Interval Graphs

Rémi Watrigant, Marin Bougeret and Rodolphe Giroudeau

LIRMM, Montpellier, France

Laboratoire d'Informatique de Robotique et de Microélectronique de Montpellier

LIRMM

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Introduction

k-Sparsest Subgraph Problem (k-SS)

Input: a graph $G = (V, E)$, $k \leq |V|$.
Output: a set $S \subseteq V$ of size exactly k.
Goal: minimize $E(S)$ (the number of edges induced by S)

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- k-SS polynomial in split graphs
- complexity of k-DS unknown in (proper) interval graphs. PTAS in interval graphs, 3-approximation in chordal graphs

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FPT Algorithm

An *FPT* algorithm for a parameterized problem is an algorithm that exactly solves the problem in $O(f(k).poly(n))$ where n is the size of the instance and k the parameter of the instance.

Polynomial-Time Approximation Scheme

A *PTAS* for a minimization problem is an algorithm $\mathcal{A}_\epsilon$ such that for any fixed $\epsilon > 0$, $\mathcal{A}_\epsilon$ runs in polynomial time and outputs a solution of cost $< (1 + \epsilon)OPT$

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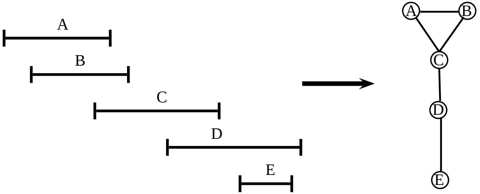

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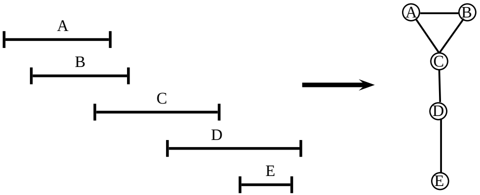

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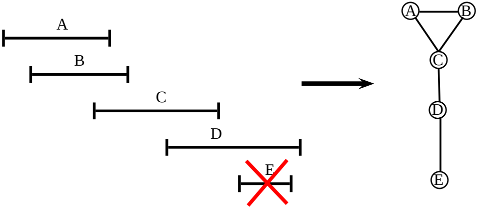

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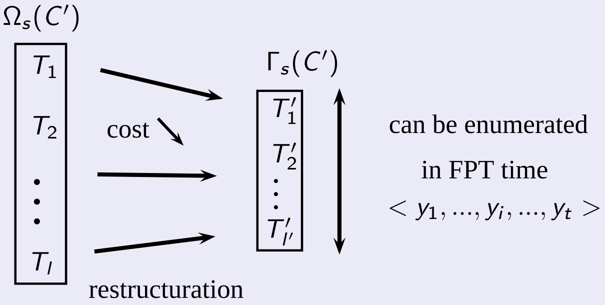
Lemma



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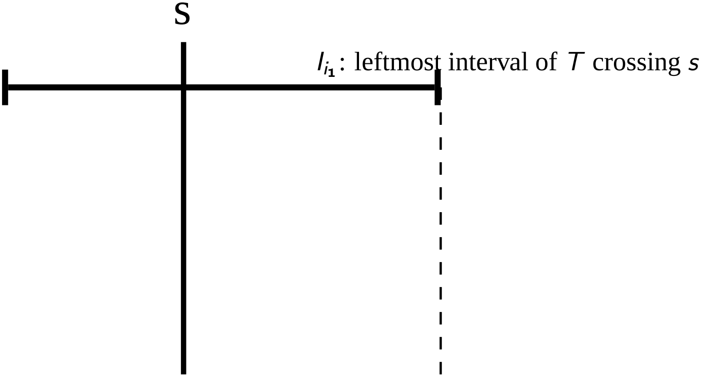

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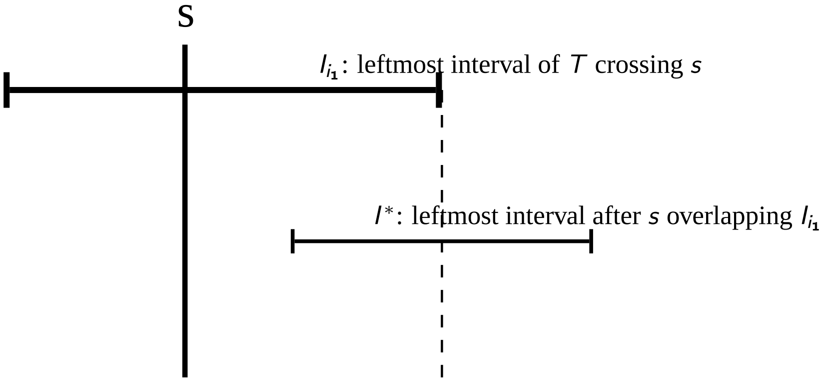

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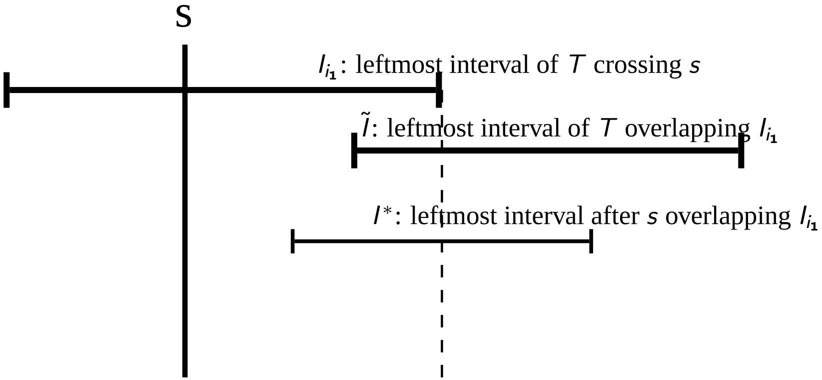

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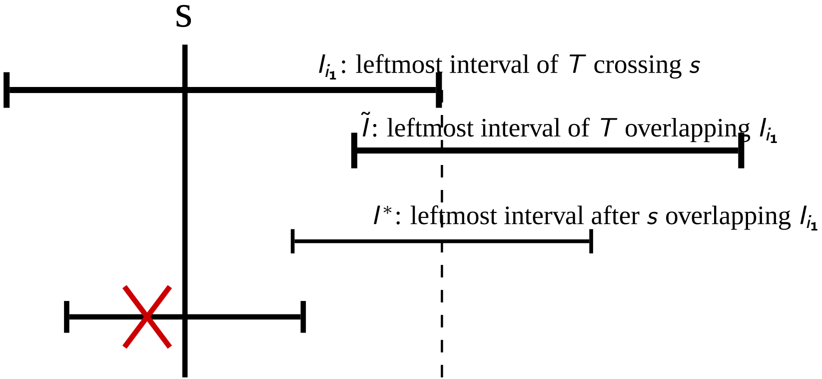

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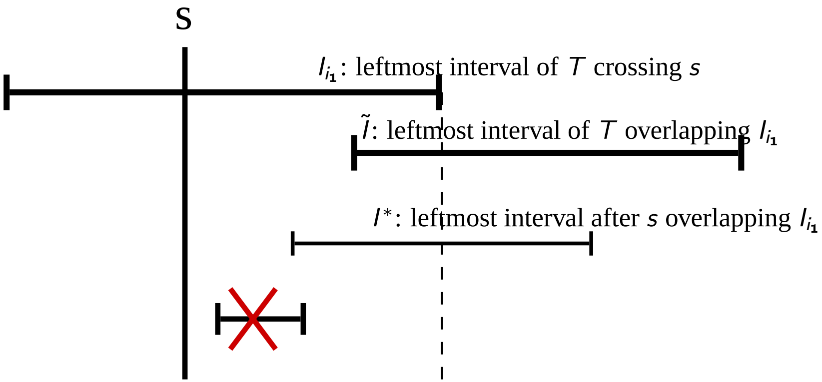

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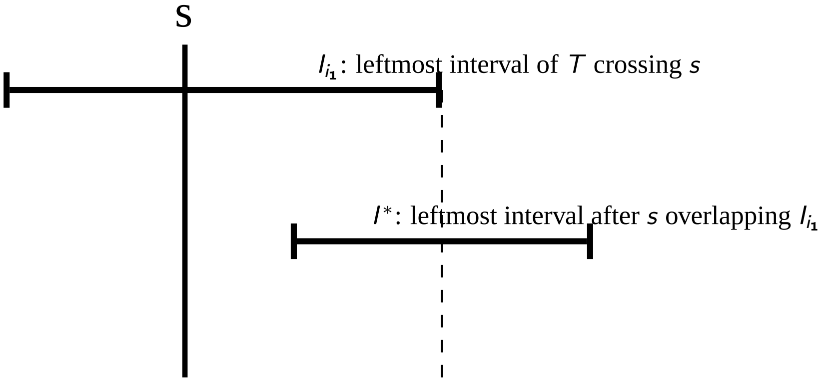

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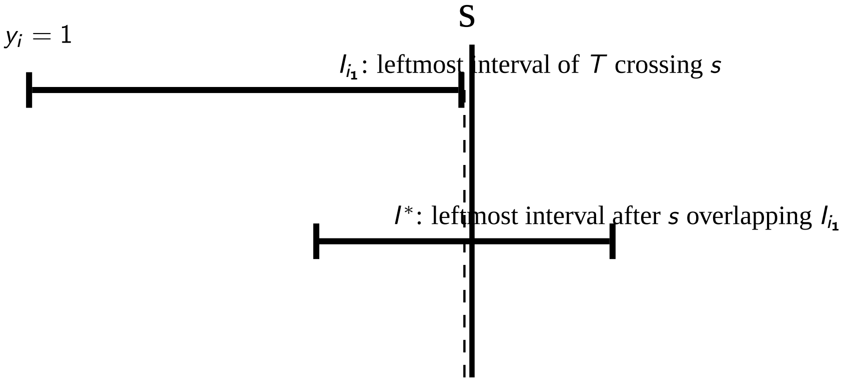

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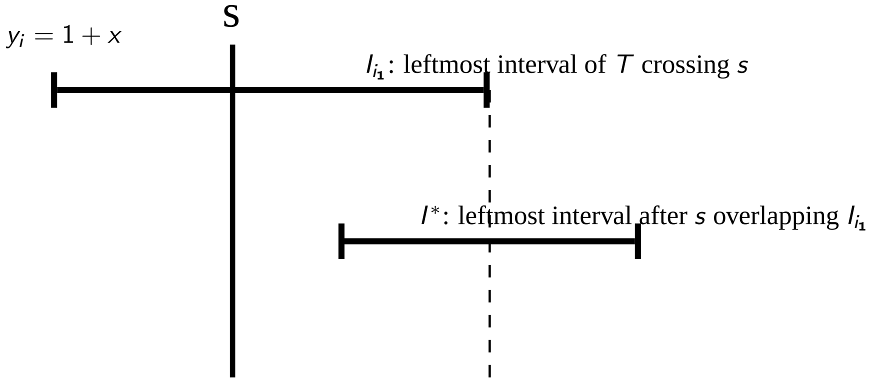

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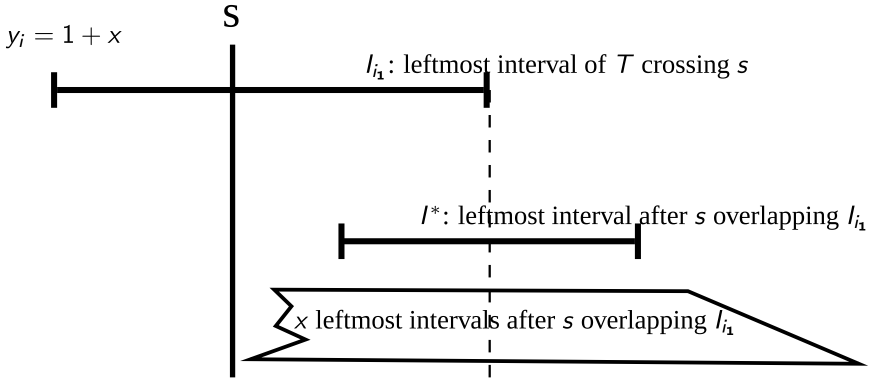

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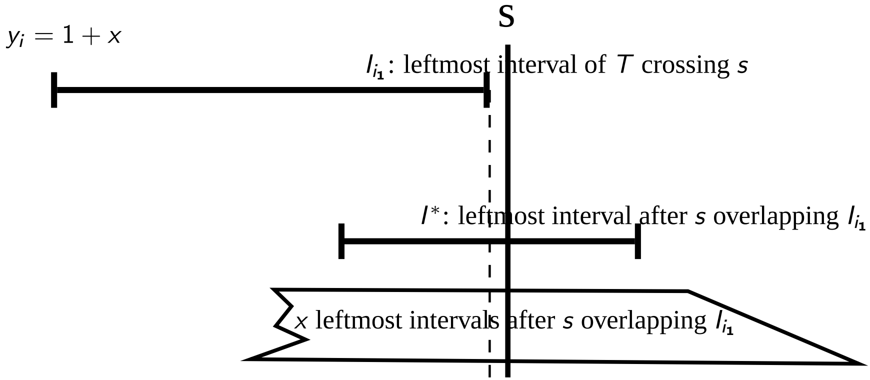

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We now bound the size of $\Gamma_s(C^*)$:

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Theorem

k-Sparsest Subgraph in Interval Graphs is FPT parameterized by the cost of the solution.

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- greedy decomposition of the graph into a path of separators
- re-structuration of an optimal solution into a near optimal solution such that all near optimal solutions can be enumerated in polynomial time
- dynamic programming processes the graph through the decomposition, enumerating all possible solutions.

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The decomposition:

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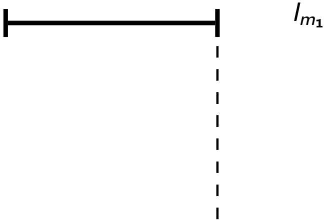

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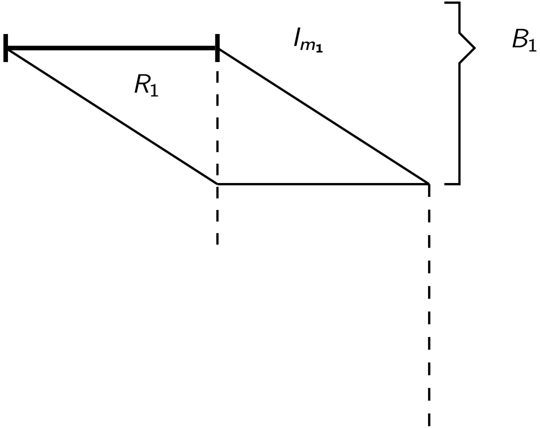

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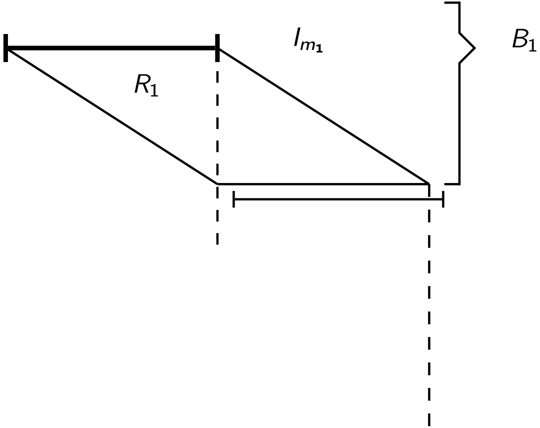

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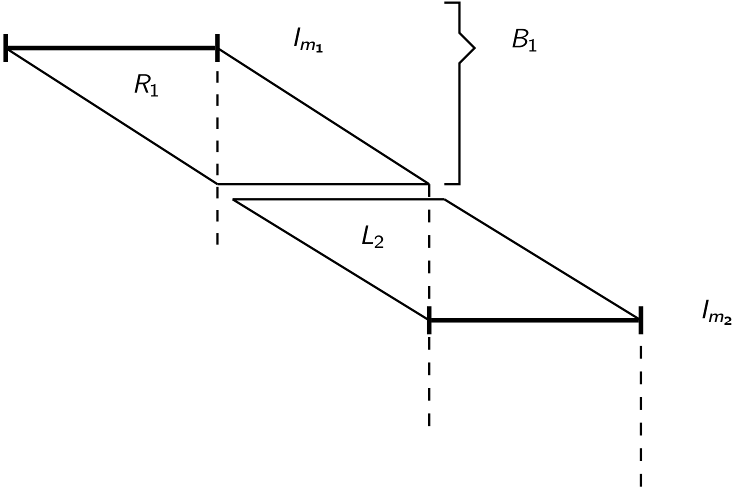

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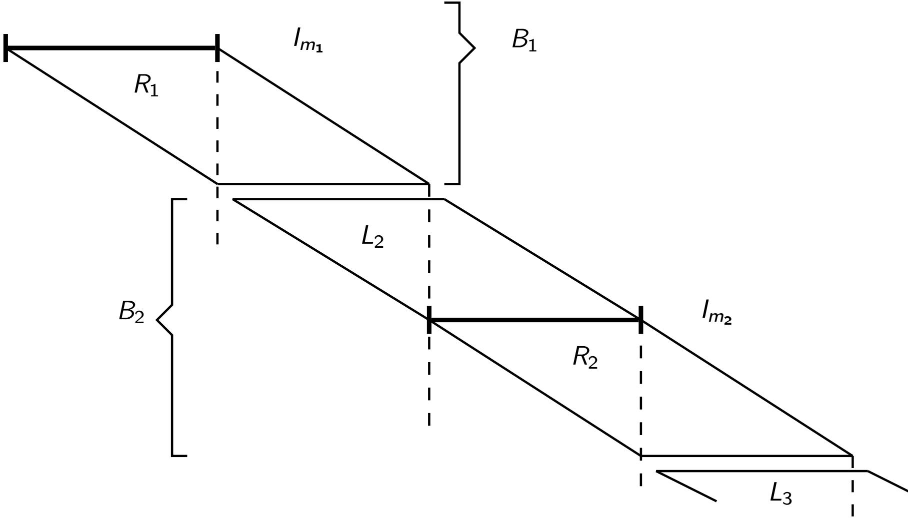

PTAS in Proper Interval Graphs

The decomposition

Remark

The only edges between blocks B_i and B_{i+1} are between R_i and L_{i+1}.
Given $S \subseteq \mathcal{I}$ we have:

$$E(S) = \sum_{i=1}^{a} E(B_i \cap S) + \sum_{i=1}^{a-1} E(R_i \cap S, L_{i+1} \cap S)$$

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Re-structuration of optimal solutions

Compaction

Let $S \subseteq \mathcal{I}$ be a solution, and $S^c = comp(S) \subseteq \mathcal{I}$ such that for each block $i \in \{1, ..., a\}$:

- for all $I \in L_i$, $comp(I) \in L_i$ and is at the right of I (we may have $comp(I) = I$)
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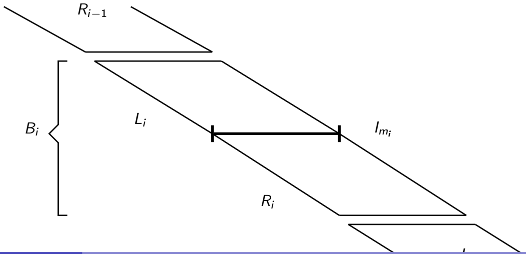

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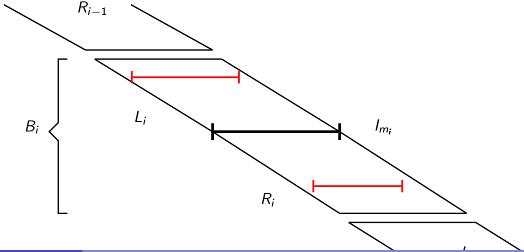

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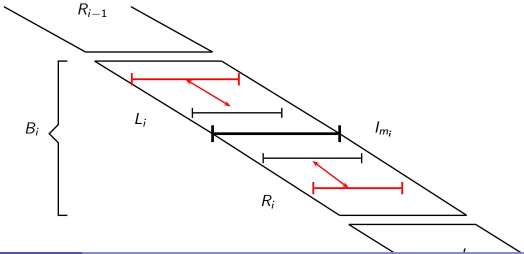

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Lemma

If *comp* is a compaction of a solution S such that for all block $i \in \{1, ..., a\}$, we have

$$E(comp(S \cap B_i)) \leq \rho E(S \cap B_i)$$

Then $comp(S)$ is a ρ-approximation of S.

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Let $X \subseteq B_i$ be a solution. We note $X = X_L \cup X_R$. Set sizes are in lowercase.

- we divide X_L into P consecutive subsets of same size $q_L \rightarrow X_1^L, ..., X_P^L$
- we divide X_R into P consecutive subsets of same size $q_R \rightarrow X_1^R, ..., X_P^R$

Then define the compaction: for any $t \in \{1, ..., P\}$

PTAS in Proper Interval Graphs

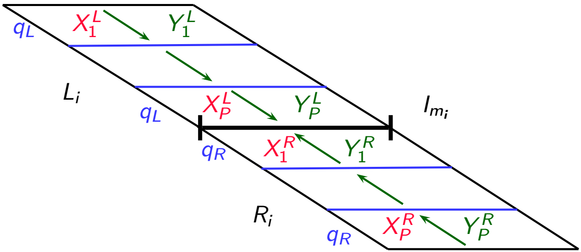
Re-structuration of optimal solutions

Let us built a compaction that yields a $(1 + \frac{4}{P})$-approximation for any fixed P.
Let $X \subseteq B_i$ be a solution. We note $X = X_L \cup X_R$. Set sizes are in lowercase.

- we divide X_L into P consecutive subsets of same size $q_L \rightarrow X_1^L, ..., X_P^L$
- we divide X_R into P consecutive subsets of same size $q_R \rightarrow X_1^R, ..., X_P^R$

Then define the compaction: for any $t \in \{1, ..., P\}$

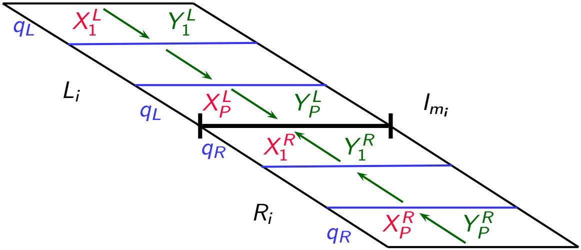
- Y_t^L are the q_L rightmost intervals at the left of the rightmost interval of X_t^L
- Y_t^R are the q_R leftmost intervals at the right of the leftmost interval of X_t^R



PTAS in Proper Interval Graphs

Re-structuration of optimal solutions

What do we need to construct such a solution ?



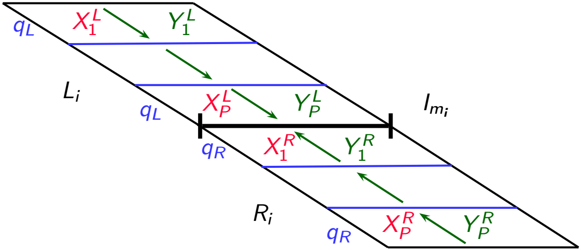
PTAS in Proper Interval Graphs

Re-structuration of optimal solutions

What do we need to construct such a solution ?

- the leftmost interval of X_t^L for $t \in \{1, ..., P\}$
- the rightmost interval of X_t^R for $t \in \{1, ..., P\}$
- x_R, x_L (plus remainders of divisions by P...)

$\Rightarrow 2P + O(1)$ variables ranging in $\{0, ..., n\}$



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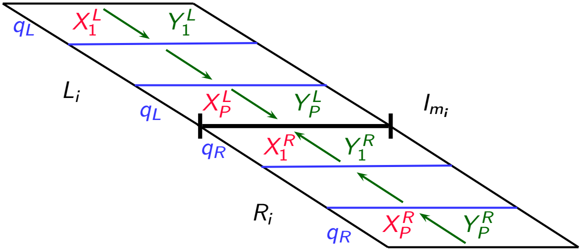

Sketch of proof of the $(1 + \frac{4}{P})$ approximation ratio:

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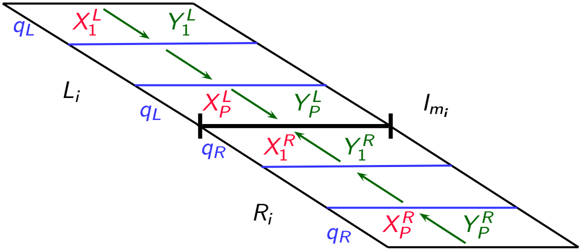
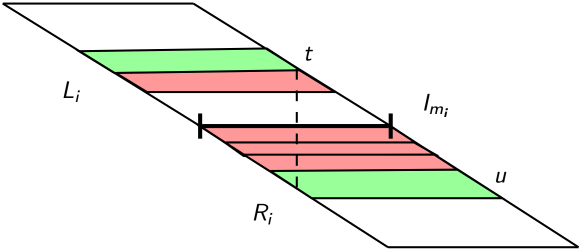


Sketch of proof of the $(1 + \frac{4}{P})$ approximation ratio:

- $OPT = \binom{x_L}{2} + \binom{x_R}{2} + \sum_{t=1}^{a} \sum_{u=1}^{a} E(X_t^L, X_u^R)$
- $SOL = \binom{x_L}{2} + \binom{x_R}{2} + \sum_{t=1}^{a} \sum_{u=1}^{a} E(Y_t^L, Y_u^R)$

PTAS in Proper Interval Graphs



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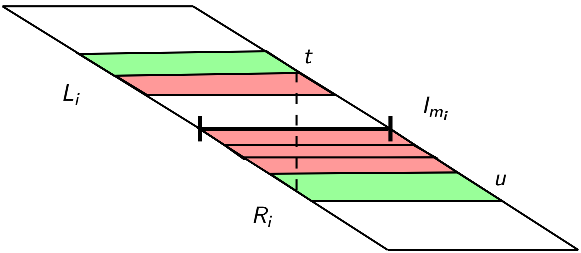
But:

PTAS in Proper Interval Graphs



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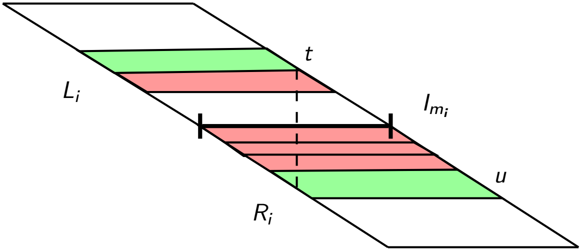
But:

- if some intervals of Y_t^L overlap some intervals of Y_u^R

Then:

- all intervals of X_{t+1}^L overlap all intervals of $\bigcup_{i=1}^{u-1} X_i^R$

PTAS in Proper Interval Graphs



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But:

- if some intervals of Y_t^L overlap some intervals of Y_u^R

Then:

- all intervals of X_{t+1}^L overlap all intervals of $\bigcup_{i=1}^{u-1} X_i^R$

Finally, we can prove that $\frac{SOL}{OPT} \leq 1 + \frac{4}{P}$

PTAS in Proper Interval Graphs

Conclusion:

Theorem

For any P, the previous algorithm outputs a $(1 + \frac{4}{P})$-approximation for the k-Sparsest Subgraph in Proper Interval graphs in $O(n^{O(P)})$

Contents

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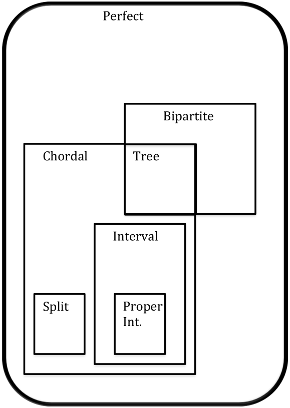

Open problems and Future Work

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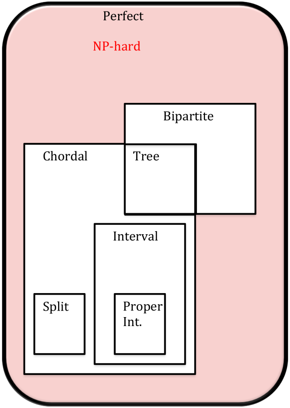

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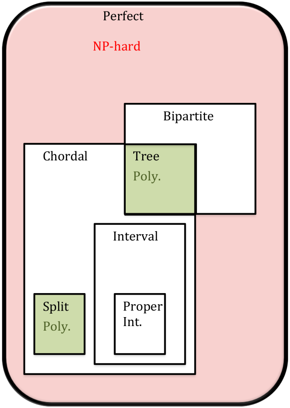

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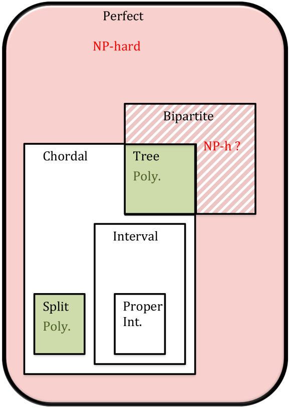

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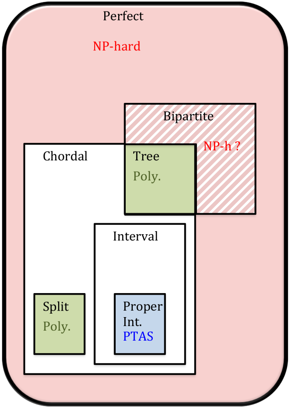

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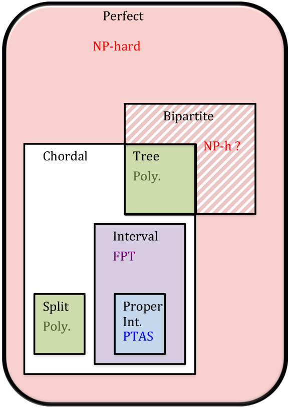

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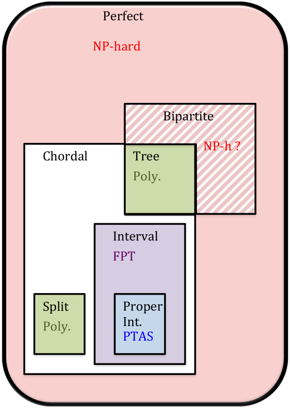

2 main objectives:

- extend FPT and/or approximation results to Chordal graphs
- NP-hardness for Chordal graphs

Thank you for your attention!